H.w. #8

pages 181-183

5) Let $\left.\begin{array}{l} u = g(x) = \sin x \\ y = f(u) = \sqrt{u} \end{array}\right\}$ $\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx} = \dfrac{1}{2} u^{-1/2} \cdot \cos x = \dfrac{\cos x}{2\sqrt{u}} = \dfrac{\cos x}{2\sqrt{\sin x}}$

6) Let $\left.\begin{array}{l} u = g(x) = \sqrt{x} \\ y = f(u) = \sin u \end{array}\right\}$ $\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx} = (\cos u) \cdot \left(\frac{1}{2} x^{-1/2}\right) = \dfrac{\cos u}{2\sqrt{x}} = \dfrac{\cos\sqrt{x}}{2\sqrt{x}}$

10) $f(x) = (1+x^4)^{2/3} \Rightarrow f'(x) = \dfrac{2}{3} (1+x^4)^{(2/3)-1} \cdot (4x^3) = \dfrac{2}{3}(1+x^4)^{-1/3} \cdot (4x^3)$

$$= \frac{8x^3}{3\sqrt[3]{1+x^4}}$$

18) $h(t) = (t^4-1)^3 \cdot (t^3+1)^4 \Rightarrow h'(t) = (t^4-1)^3 \cdot 4 \cdot (t^3+1) \cdot (3t^2) + 3 \cdot 4(t^3) \cdot (t^4-1)^2 \cdot (t^3+1)^4$

$$= 12t^2 (t^4-1)^2 (t^3+1)^3 (2t^4+t-1)$$

24) $f(x) = \dfrac{x}{\sqrt{7-3x}} \Rightarrow f'(x) = \dfrac{1 \cdot \sqrt{7-3x} - x \cdot \frac{1}{2}(7-3x)^{-1/2} \cdot (-3)}{\left(\sqrt{7-3x}\right)^2}$

$$= \frac{\sqrt{7-3x} + \frac{3x}{2\sqrt{7-3x}}}{7-3x}$$

32) $y = \tan^2(3\theta) = (\tan(3\theta))^2 \Rightarrow y' = 2 \cdot (\tan 3\theta) \cdot \dfrac{d}{d\theta}(\tan 3\theta) = 2 \cdot \tan 3\theta \cdot \sec^2 3\theta \cdot 3$

$$= 6 \cdot \tan 3\theta \cdot \sec^2 3\theta$$

38) $y = \sin(\sin(\sin x)) \Rightarrow y' = \cos(\sin(\sin x)) \dfrac{d}{dx}(\sin(\sin x))$

$= \cos(\sin(\sin x)) \,.\, \cos(\sin x) \,.\, \cos x$

42) $y = \sqrt{\cos(\sin^2 x)} \Rightarrow y' = \dfrac{1}{2}\left(\cos(\sin^2 x)\right)^{-1/2} . \left[-\sin(\sin^2 x)\right] . (2\sin x . \cos x)$

$= -\dfrac{\sin(\sin^2 x) . \sin x . \cos x}{\sqrt{\cos(\sin^2 x)}}$

pages 188–190

6) $\dfrac{d}{dx}(x^2 - y^2) = \dfrac{d}{dx}(1) \Rightarrow 2x - 2y\,y' = 0 \Rightarrow 2y\,y' = 2x \Rightarrow y' = \dfrac{\cancel{2}\,x}{\cancel{2}\,y} = \dfrac{x}{y}$

8) $\dfrac{d}{dx}(x^2 - 2xy + y^3) = \dfrac{d}{dx}(c) \Rightarrow 2x - 2(x.y' + y.1) + 3.y^2.y' = 0$

$\Rightarrow 2x - 2x.y' - 2y + 3y^2.y' = 0$

$\Rightarrow 2x - 2y = 2x.y' - 3y^2.y'$

$\Rightarrow 2x - 2y = (2x - 3y^2).y'$

$\Rightarrow \dfrac{2x - 2y}{2x - 3y^2} = y'$

12) $\dfrac{d}{dx}(1+x) = \dfrac{d}{dx}\left[\sin(x.y^2)\right] \Rightarrow 1 = \cos(x.y^2)\,(x.2y.y' + y^2.1)$

$\Rightarrow 1 = \cos(x.y^2).(2x.y.y') + \cos(x.y^2).y^2$

$\Rightarrow 1 - y^2.\cos(x.y^2) = 2x.y.y'.\cos(x.y^2)$

$\Rightarrow \dfrac{1 - y^2.\cos(x.y^2)}{2x.y.\cos(x.y^2)} = y'$

28) $x^{2/3} + y^{2/3} = 4 \Rightarrow \frac{2}{3} x^{-1/3} + \frac{2}{3} y^{-1/3} \cdot y' = 0$

$$\Rightarrow \frac{1}{\sqrt[3]{x}} + \frac{y'}{\sqrt[3]{y}} = 0$$

$$\Rightarrow y' = -\frac{\sqrt[3]{y}}{\sqrt[3]{x}}$$

when $x = -3\sqrt{3}$ and $y = 1$, we have $y' = -\frac{1}{(-3\sqrt{3})^{1/3}} = -\frac{(-3\sqrt{3})^{2/3}}{-3\sqrt{3}} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$

so the equation of the tangent line is $y - 1 = \frac{1}{\sqrt{3}}(x + 3\sqrt{3})$ or $y = \frac{1}{\sqrt{3}}x + 4$

36) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{2x}{a^2} + \frac{2y \cdot y'}{b^2} = 0 \Rightarrow y' = -\frac{b^2 \cdot x}{a^2 \cdot y}$

An equation of the tangent line at (x_0, y_0) is $y - y_0 = \frac{b^2 \cdot x_0}{a^2 \cdot y_0}(x - x_0)$

Multiplying both sides by $\frac{y_0}{b^2}$ gives $\frac{y_0 \cdot y}{b^2} - \frac{y_0^2}{b^2} = \frac{x_0 \cdot x}{a^2} - \frac{x_0^2}{a^2}$

Since (x_0, y_0) lies on the hyperbola, we have $\frac{x_0 x}{a^2} - \frac{y_0 y}{b^2} = \frac{x_0^2}{a^2} - \frac{y_0^2}{b^2} = 1.$

40) $y^q = x^p \Rightarrow q \cdot y^{q-1} \cdot y' = p \cdot x^{p-1} \Rightarrow y' = \frac{p \cdot x^{p-1}}{q \cdot y^{q-1}} = \frac{p \cdot x^{p-1} \cdot y}{q \cdot y^q} = \frac{p \cdot x^{p-1} \cdot x^{p/q}}{q \cdot x^p} = \frac{p}{q} x^{(p/q) - 1}$

42) $x^2 - y^2 = 5$ and $4x^2 + 9y^2 = 72$ intersect when $4x^2 + 9(x^2 - 5) = 72$

$$\Leftrightarrow 4x^2 + 9x^2 - 45 = 72$$

$$\Leftrightarrow 13x^2 = 117$$

$$\Leftrightarrow x = \pm 3$$

so there are four points of intersection $(\pm 3, \pm 2)$. $x^2 - y^2 = 5 \Rightarrow 2x - 2yy' = 0$

$$\Rightarrow y' = \frac{x}{y}$$

and $4x^2 + 9y^2 = 72 \Rightarrow 8x + 18y \cdot y' = 0 \Leftrightarrow y' = -4x/(9y)$. At $(3, 2)$, the slopes $m_1 = \frac{3}{2}$ and $m_2 = \frac{-2}{3}$, so the curves are orthogonal. By symmetry, the curves are also orthogonal at $(3, -2)$, $(-3, 2)$ and $(-3, -2)$.