Math 2433-001

	Circle	Ellipse	Parabola	Hyperbola
Cone-plane intersection description	Plane's normal is parallel to the cone axis	Plane's normal makes makes a positive angle with cone axis. Angle is less than cone angle.	Plane's normal makes an angle with the cone axis which is equal to cone angle.	Plane's normal makes an angle with cone axis which is greater than the cone angle.
Has a focus-locus description	YES	YES	NO	YES
	PF = C	$ PF_1 + PF_2 = C$		$ PF_1 - PF_2 = C$
Has a focus-directrix description	NO	YES	YES	YES
		$ PF_1 = \epsilon PD $ $0 < \epsilon < 1$	$ PF_1 = PD $	$\begin{aligned} PF_1 &= \epsilon PD \\ \epsilon &> 1 \end{aligned}$
Standard cartesian description	$x^2 + y^2 = a^2$	$(x/a)^2 + (y/b)^2 = 1$ $a \ge b > 0 \text{ or}$ $b \ge a > 0$	$x^{2} = \pm 4py$ or $y^{2} = \pm 4px$	$(x/a)^2 - (y/b)^2 = 1$ or $(y/b)^2 - (x/a)^2 = 1$
Standard polar description	$r = l/(1 \pm \epsilon \cos(\theta))$ or $r = l/(1 \pm \epsilon \sin(\theta))$ with $\epsilon = 0$	with $0 < \epsilon < 1$	with $\epsilon = 1$	with $\epsilon > 1$

Standard Cartesian Conics.

Note that for the ellipse $x^2/a^2 + y^2/b^2 = 1$ (with a > b) we have foci at $(\pm c, 0)$ where $c^2 = a^2 - b^2$. For the hyperbola $x^2/a^2 - y^2/b^2 = 1$ we have focus at $(\pm c, 0)$ where $c^2 = a^2 + b^2$. Asymptotes are $y = \pm (b/a)x$. For parabola $x^2 = 4py$ we have directrix at y = -p and focus at (0, p). Other standard equations are obtained by rotating these examples through $\pi/2$, π and $3\pi/2$ radians.

Standard Polar Conics.

For $\epsilon > 0$ the equation $r = l/(1 + \epsilon \cos(\theta))$ has one focus at the origin, and directrix at x = d. Here d is found from the equation $\epsilon d = l$.

The 3 other standard polar equations arise by taking $\pi/2$, π and $3\pi/2$ rotations of this conic.

Polar Coordinates. Arclength $L = \int_{\alpha}^{\beta} \sqrt{r^2 + (dr/d\theta)^2} d\theta$. Area $A = (1/2) \int_{\alpha}^{\beta} r^2 d\theta$.

Parametric Curves. Arclength $L = \int_{a}^{b} \sqrt{(dx/dt)^{2} + (dy/dt)^{2}} dt$.