MATH 2513-001 **QUIZ III** **NAME:** SOLUTIONS

Tuesday, October 13, 2009

Q1]... Define what it means for a function $f : A \to B$ to be *injective.*

$f : A \to B$ is injective if $\forall\, a_1, a_2 \in A$,

$f(a_1) = f(a_2) \implies a_1 = a_2$,

Define what it means for a function $f : A \to B$ to be *surjective.*

$f : A \to B$ is surjective if

$\forall b \in B \quad \exists\, a \in A$ such that $f(a) = b$.

Say whether each of the following functions are *injective*, *surjective* or both.

(1) $f : \mathbb{Z} \to \mathbb{Z} : n \mapsto 10n$

$f(n) = f(m) \Rightarrow 10n = 10m \Rightarrow \frac{\cancel{10}n}{\cancel{10}} = \frac{\cancel{10}m}{\cancel{10}} \Rightarrow n = m$ $\Rightarrow$ $\boxed{f \text{ injective}}$ ✓

f is not surjective

eg $f(x) = 1 \Rightarrow 10x = 1 \Rightarrow x = \frac{1}{10} \notin \mathbb{Z}$

(2) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} : (m, n) \mapsto m + n$

$\boxed{f \text{ surjective}}$ $\forall\, n \in \mathbb{Z}$,

$(0,n) \in \mathbb{Z}\times\mathbb{Z}$ and $f(0,n) = 0+n = n$.

$\Rightarrow f$ onto

f not injective

$f(-1,1) = -1+1 = 0 = -2+2 = f(-2,2)$

but $(-1,1) \neq (-2,2)$.

(3) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z} : (m, n) \mapsto (3m + n, m + n)$

$\boxed{f \text{ injective}}$

$f(m,n) = f(a,b)$

$\Rightarrow (3m+n,\ m+n) = (3a+b,\ a+b)$

$\Rightarrow 3m+n = 3a+b$

subtract $m+n = a+b$

$2m = 2a$

$\Rightarrow m = a \rightarrow m+n = a+b,\ m = a \Rightarrow n = b$

$\Rightarrow (m,n) = (a,b)$

f not surjective

eg. $(1,0)$ is not in range of f.

$f(m,n) = (1,0)$

$\Rightarrow 3m+n = 1$

$m+n = 0$

$\Rightarrow 2m = 1$

$\Rightarrow m = \frac{1}{2} \notin \mathbb{Z}$.

Other example (3)

$$f: \mathbb{Z} \times \mathbb{Z} \longrightarrow \mathbb{Z} \times \mathbb{Z}$$
$$: (m,n) \longmapsto (2m+n,\ m+n)$$

f is injective (proof same as previous eg).

? f surjective ? $\rightsquigarrow$ $f(m,n) = (x,y)$

$$2m+n = x$$
$$m+n = y$$
$$\overline{m = x-y}$$

$$\Rightarrow (x-y) + n = y$$
$$\Rightarrow n = 2y - x$$

So $f(x-y,\ 2y-x) = (x,y)$

& clearly $(x-y,\ 2y-x) \in \mathbb{Z} \times \mathbb{Z}$

so f is surjective

$\Rightarrow$ f is a bijection!