Fa'15: MATH 2924–010 **Differential and Integral Calculus II** **Noel Brady**

Friday 11/20/2015 Makeup Midterm III 50 minutes

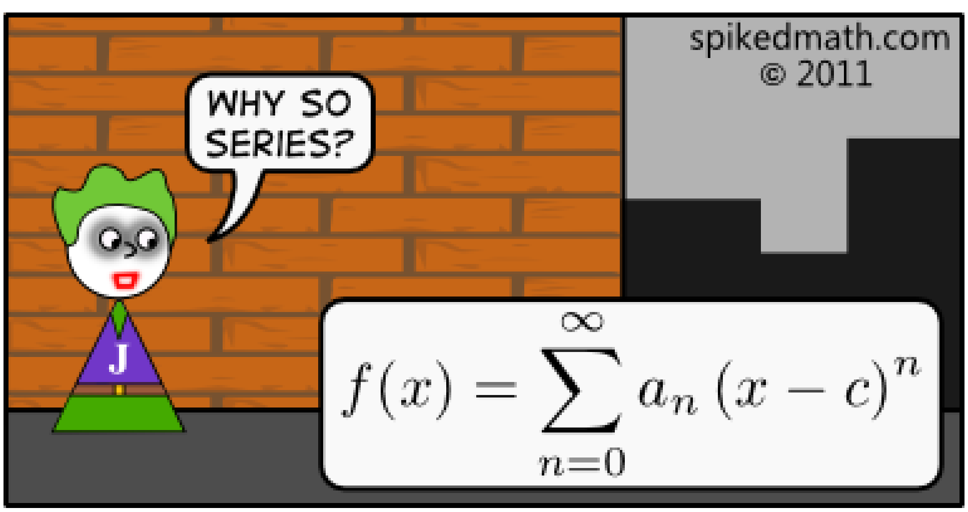
Name:

Student ID:

Instructions.

1. Attempt all questions.
2. Do not write on back of exam sheets. Extra paper is available if you need it.
3. Show all the steps of your work clearly.

Question	Points	Your Score
Q1	20	
Q2	20	
Q3	20	
Q4	20	
Q5	20	
TOTAL	100	



Some results about series.

1. **Geometric Series.**
$\sum_{n=1}^{\infty} ar^{n-1}$ converges when $|r| < 1$; it converges to the sum $\frac{a}{1-r}$ when $|r| < 1$.

2. **Test for Divergence.**
If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

3. **Integral Test.**
For $f(x)$ continuous on $[1, \infty)$, positive and decreasing to 0, the series $\sum_{n=1}^{\infty} f(n)$ converges if and only if the improper integral $\int_1^{\infty} f(x)dx$ converges.

4. **Comparison Tests.**
Direct comparison test: compares series of positive terms, term-by-term.

   Limit comparison test: compares series of positive terms $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ when $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ a finite limit not equal to 0.

5. **Root Test.**
Let $\lim_{n\to\infty} |a_n|^{1/n} = L$. If $L < 1$ then $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, and if $L > 1$ then it is divergent.

6. **Ratio Test.**
Let $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = L$. If $L < 1$ then $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, and if $L > 1$ then it is divergent.

7. **Alternating Series Test.**
If a_n are positive, decreasing to 0, then $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ is convergent. Moreover, the nth partial sum is within a_{n+1} of the sum of the whole series.

8. **Power series.**
Ratio test is useful for computing the radius of convergence of a power series $\sum_{n=0}^{\infty} c_n (x-a)^n$.

9. **Taylor and Maclaurin Series.**
Taylor series for $f(x)$ centered about a is given by

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

   Maclaurin series for $f(x)$ is the Taylor series for $f(x)$ centered about 0.

10. **Remainder Estimate.**
Taylor's inequality states that if $|f^{(n+1)}(x)| \leq M$ on the interval $[a-d, a+d]$, then

$$|f(x) - T_n(x)| \ \leq \ \frac{M|x-a|^{(n+1)}}{(n+1)!}$$

    on the interval $[a-d, a+d]$. Here $T_n(x)$ is the *degree n Taylor polynomial approximation* to $f(x)$.

Q1]. . . [20 points] Test the following series for convergence or divergence. Show all the steps of your work.

$$\sum_{n=1}^{\infty} \frac{1(3)(5)\cdots(2n-1)}{5^n n!}$$

Q2]. . . [20 points] Determine whether the following series converges absolutely, converges conditionally, or diverges. Show all the steps of your work.

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{7n+1}}$$

Q3]... [20 points] Determine the radius of convergence and the interval of convergence of the following power series. Show all the steps of your work.

$$\sum_{n=1}^{\infty} \frac{(5x-4)^n}{n^3}$$

Q4]. . . [20 points] Find the Taylor series for $f(x) = \cos x$ centered about $\pi/2$. Show all the steps of your work.

Q5]. . . [20 points] Find an estimate for the definite integral

$$\int_0^1 \cos(x^2)dx$$

which is within 0.01 of the actual value of this integral. Show all the steps of your work.