PRINT NAME: SOLUTIONS

Calculus I [1823–001] Quiz I

Q1]... Find the absolute maximum and absolute minimum values of the function

$$f(x) = \sqrt{8 - x^2 + 2x}$$

on the interval $[0, 3]$.

Solution. We have two endpoints: 0 and 3.

The critical points are found by solving $f' = 0$ or finding where f' doesn't exist. Well,

$$f'(x) = \frac{-2x + 2}{2\sqrt{8 - x^2 + 2x}} = -\frac{x - 1}{\sqrt{8 - x^2 + 2x}}$$

Note that $f'(x)$ is defined everywhere on the interval $[0, 3]$, so the critical points will be where $f' = 0$. Now, $f' = 0$ only when the numerator is zero. Thus $f' = 0$ only when $x - 1 = 0$, or $x = 1$.

So the points we have to evaluate are: 0, 1, and 3. We have

$$\begin{aligned} f(0) &= \sqrt{8} \\ f(1) &= \sqrt{9} = 3 \\ f(3) &= \sqrt{5} \end{aligned}$$

Therefore, the absolute maximum is 3, and this occurs at the point 1. The absolute minimum is $\sqrt{5}$, and this occurs at the point 3.

Finally, note that this is just an arc of a circle graph!! since $8 - x^2 + 2x = 3^2 - (x - 1)^2$. Thus, $(x - 1)^2 + y^2 = 3^2$, which is a circle of radius 3 and center $(1, 0)$. Now our answers make geometric sense.