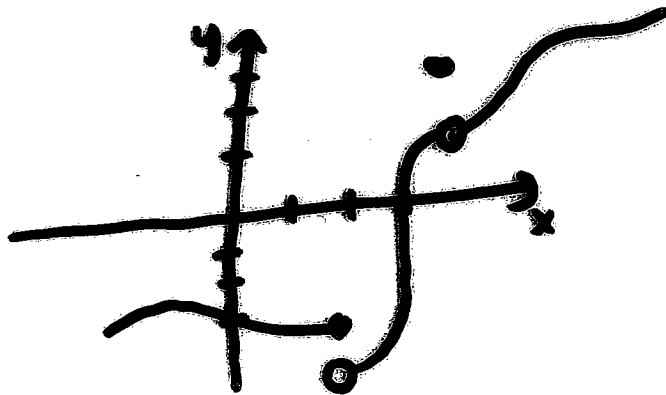


HW #4

①

pages 110-112

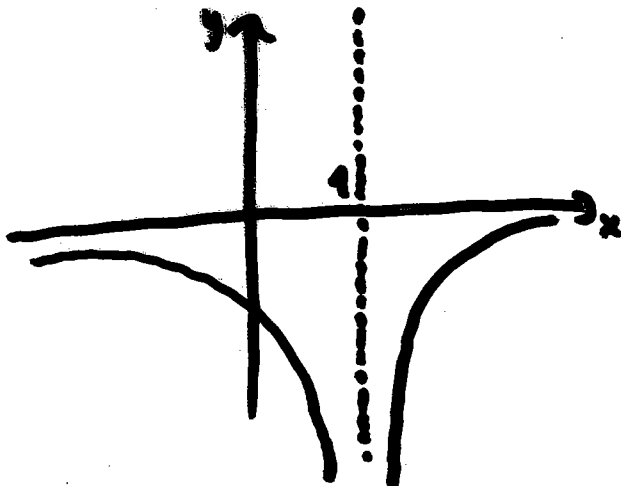
6) The graph can be shown as;



(The answer vary)

15) $f(x) = -\frac{1}{(x-1)^2}$ is discontinuous at 1

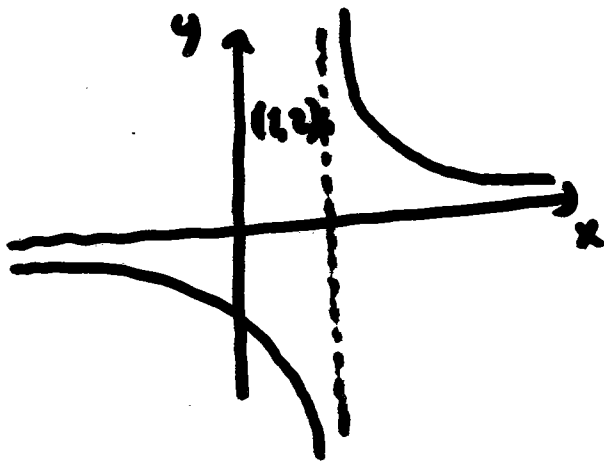
since $f(1)$ is not defined.



(2)

$$16) f(x) = \begin{cases} \frac{1}{x-1} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases} \text{ is discontinuous}$$

at 1 because $\lim_{x \rightarrow 1} f(x)$ does not exist.



$$\lim_{x \rightarrow 1^+} f(x) = \infty$$

$$\lim_{x \rightarrow 1^-} f(x) = -\infty$$

$$20) f(x) = \begin{cases} 1+x^2 & \text{if } x < 1 \\ 4-x & \text{if } x \geq 1 \end{cases}$$

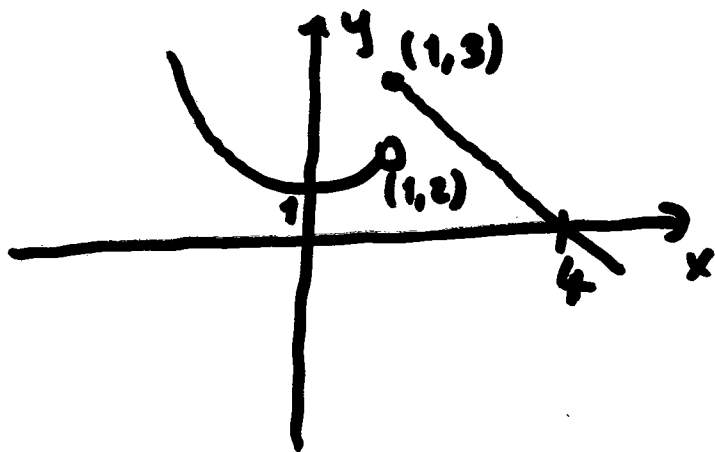
$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (1+x^2) = 1+1=2$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (4-x) = 4-1=3$$

Thus, f is discontinuous at 1 because

3

$\lim_{x \rightarrow 1} f(x)$ does not exist.



24) By theorem 7, the trigonometric function $\sin x$ and the polynomial function $x+1$ are continuous on \mathbb{R} . By part 5 of theorem 4, $h(x) = \frac{\sin x}{x+1}$ is continuous on its domain

$$\{x \mid x \neq -1\}$$

28) The sine and cosine functions are continuous everywhere by theorem 7 so

$$F(x) = \sin(\cos(\sin x))$$

is continuous everywhere by theorem 9.

④

43) $f(x) = x^3 - x^2 + x$ is continuous on the interval $[2, 3]$ since it is a polynomial.

$$f(2) = 6 \quad \text{and} \quad f(3) = 21.$$

Since $6 < 10 < 21$, there is a number c in $(2, 3)$ such that $f(c) = 10$ by Intermediate Value Theorem

59) If there is such a number, it satisfies the equation $x^3 + 1 = x$

$$\Leftrightarrow x^3 - x + 1 = 0$$

$$\text{Let } f(x) = x^3 - x + 1$$

$f(x)$ is continuous since it is a polynomial and $f(-2) = -5 < 0$ and $f(-1) = 1 > 0$. Now by Intermediate Value theorem, there is a number c between -2 and -1 such

5

that $f(c)=0$, so that $c^3+1=c$.

pages 119-121

8) Using $\lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0} = m$,

$$m = \lim_{x \rightarrow 4} \frac{\sqrt{2x+1} - \sqrt{2 \cdot 4 + 1}}{x - 4} = \lim_{x \rightarrow 4} \frac{\sqrt{2x+1} - 3}{x - 4}$$

$$= \lim_{x \rightarrow 4} \frac{\sqrt{2x+1} - 3}{x - 4} \cdot \frac{(\sqrt{2x+1} + 3)}{(\sqrt{2x+1} + 3)}$$

$$= \lim_{x \rightarrow 4} \frac{2x+1 - 3^2}{(x-4)(\sqrt{2x+1} + 3)}$$

$$= \lim_{x \rightarrow 4} \frac{2(x-4)}{(x-4)(\sqrt{2x+1} + 3)} = \frac{2}{\sqrt{2 \cdot 4 + 1} + 3}$$

$$= \frac{1}{3}$$

Tangent line: $y - 3 = \frac{1}{3}(x - 4)$

$$y = \frac{1}{3}(x - 4) + 3$$

15) a) Since the slope of the tangent at $t=0$ is 0, the car's initial velocity was 0.

b) The slope of the tangent is greater at C than at B, so the car was going faster at C.

c) Near A, the tangent lines are becoming steeper as x increases, so the velocity was increasing, so the car was speeding up.

Near B, the tangent lines are becoming less steep, so the car was slowing down.

The steepest tangent near C is the one at C, so at C the car had just finished speeding up, and was about to start slowing down.

