H.W. #7

3.5 Derivatives of Trigonometric Functions

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4) $y = 2\csc x + 5\cos x$

$y' = -2\csc x \cdot \cot x - 5\sin x$

6) $g(t) = 4\sec t + \tan t$

$g'(t) = 4\sec t \cdot \tan t + \sec^2 t$

12) $y = \dfrac{\tan x - 1}{\sec x}$

$$y' = \frac{(\tan x - 1)' \cdot \sec x - (\sec x)' \cdot (\tan x - 1)}{(\sec x)^2}$$

$$= \frac{\sec^2 x \cdot \sec x - \sec x \cdot \tan x \cdot (\tan x - 1)}{(\sec x)^2}$$

$$= \frac{\cancel{\sec x}\,(\sec^2 x - \tan x(\tan x - 1))}{(\sec x)^{\cancel{2}}}$$

$$= \frac{\sec^2 x - \tan^2 x + \tan x}{\sec x}$$

16) Differentiate $y = x.\sin x.\cos x$.

Remember if $y = f(x).g(x).h(x)$ then

$$y' = f'(x).g(x).h(x) + f(x).g'(x).h(x) + f(x).g(x).h'(x)$$

By applying this formula, we have

$$y' = x'.\sin x.\cos x + x.(\sin x)'.\cos x + x.\sin x.(\cos x)'$$

$$= 1.\sin x.\cos x + x.\cos x.\cos x + x.\sin x.(-\sin x)$$

$$= \sin x.\cos x + x.\cos^2 x - x.\sin^2 x$$

22) Find an equation of the tangent line to the curve at the given point.

$y = (1+x).\cos x$, $(0,1)$

$$y' = (1+x)'.\cos x + (1+x).(\cos x)'$$

$$= \cos x + (1+x).(-\sin x)$$

$$m = y'(0) = \cos 0 + (1+0).(-\sin 0)$$

$$= 1$$

hence the tangent line equation is

$$y-1=1.(x-0)$$
$$y=x+1$$

24) $y=\dfrac{1}{\sin x+\cos x}$, $(0,1)$. Tangent line = ?

$$y'=-\frac{\cos x-\sin x}{(\sin x+\cos x)^2}$$

$$m=y'(0)=-\frac{\cos 0-\sin 0}{(\sin 0+\cos 0)^2}=-1$$

Tangent line $\Rightarrow$ $y-1=-1.(x-0)$
$$y=-x+1$$

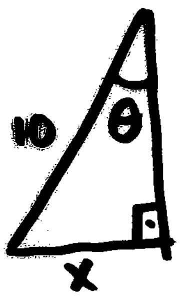
31) a) $x(t)=8.\sin t \Rightarrow v(t)=x'(t)=8.\cos t$

b) The mass at time $t=\frac{2\pi}{3}$ has position

$$x\left(\frac{2\pi}{3}\right)=8.\sin\left(\frac{2\pi}{3}\right)=8.\frac{\sqrt{3}}{2}=4\sqrt{3}$$
$$v\left(\frac{2\pi}{3}\right)=8.\cos\left(\frac{2\pi}{3}\right)=8.\left(\frac{-1}{2}\right)=-4$$

since $v\left(\frac{2\pi}{3}\right)<0$, the particle is moving to the left.

33)



$$\sin\theta = \frac{x}{10} \iff x = 10 \cdot \sin\theta$$

Rate of change of x with respect to θ is $\frac{dx}{d\theta}$.

$$\frac{dx}{d\theta} = 10 \cdot \cos\theta$$

when $\theta = \frac{\pi}{3}$, $\frac{dx}{d\theta} = 10 \cdot \cos\frac{\pi}{3} = 10 \cdot \frac{1}{2} = 5\,ft/$

36) $$\lim_{x\to 0} \frac{\sin 4x}{\sin 6x} = \lim_{x\to 0}\left(\frac{\sin 4x}{x} \cdot \frac{x}{\sin 6x}\right)$$

$$= \lim_{x\to 0}\left(4 \cdot \frac{\sin 4x}{4x} \cdot \frac{1}{6} \cdot \frac{6x}{\sin 6x}\right)$$

$$= \lim_{x\to 0} 4 \cdot \left(\frac{\sin 4x}{4x}\right) \cdot \frac{1}{6} \cdot \lim_{x\to 0} \frac{6x}{\sin 6x}$$

$$= 4 \cdot (1) \cdot \frac{1}{6} \cdot (1)$$

$$= \frac{4}{6}$$

$$= \frac{2}{3}$$

38) $$\lim_{\theta \to 0} \frac{\cos\theta - 1}{\sin\theta} = \lim_{\theta \to 0} \frac{\frac{\cos\theta - 1}{\theta}}{\frac{\sin\theta}{\theta}} = \frac{\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta}}{\lim_{\theta \to 0} \frac{\sin\theta}{\theta}}$$

$$= \frac{0}{1} = 0.$$

40) $$\lim_{t \to 0} \frac{\sin^2 3t}{t^2} = \lim_{t \to 0} \left(\frac{\sin 3t}{t} \cdot \frac{\sin 3t}{t} \right)$$

$$= \lim_{t \to 0} \left(3 \cdot \frac{\sin 3t}{3t} \cdot 3 \cdot \frac{\sin 3t}{3t} \right.$$

$$= 3 \cdot 3 \cdot \lim_{t \to 0} \frac{\sin 3t}{3t} \cdot \lim_{t \to 0} \frac{\sin 3t}{3t}$$

$$= 9 \cdot 1 \cdot 1$$

$$= 9.$$