Friday 02/13/2004   Midterm I   8:30am–9:20am

Name: _____   Student ID: _____

Instructions.

1. No calculators or notes.

2. Attempt all questions.

3. Do not write on back of exam sheets. Extra paper is available.

4. Show all the steps of your work clearly.
   The method (reasoning) used to obtain an answer is worth more than the answer itself.

Question	Points	Your Score
Q1	9	
Q2	10	
Q3	8	
Q4	10	
Q5	10	
Q6	8	
TOTAL	55	

Q1]... [9 points] Evaluate the following limit, showing all your work.

$$\lim_{x \to 2} \frac{2(x-1)^2 - 2}{x - 2}$$

This limit represents the slope of the tangent line to the graph of some function at some point. Determine a suitable function? What is the point on the graph? Draw a sketch of the graph. Include the point and the tangent line.

Q2].. .[10 points] Compute the following limits. Include the details of your work.

$$\lim_{h \to 0} \frac{\sqrt{3+h} - \sqrt{3}}{h}$$

$$\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$$

Q3].…[8 points] Give the precise definition of the following limit (the version involving a notion of closeness for output values, ϵ, and a notion of closeness for input values, δ)
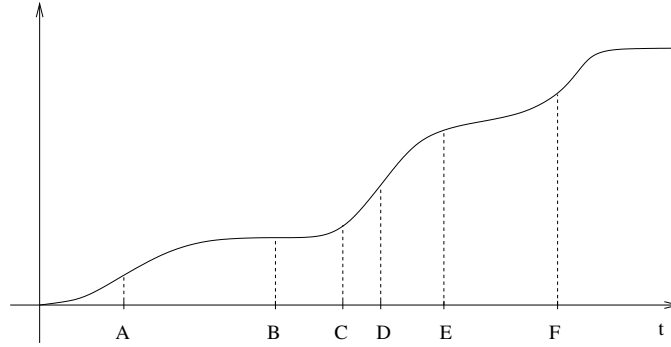
$$\lim_{x \to a} f(x) \ = \ L$$

Hint: It starts like this. *We say that $\lim_{x \to a} f(x) = L$ if for every $\epsilon > 0$, there exists.…*

Use the precise definition of a limit to show that

$$\lim_{x \to 2} 3x + 1 \ = \ 7$$

Q4].. . [10 points] The following graph represents the distance (measured in miles) that a car has traveled in time t (measured in hours).

Answer the following questions, giving reasons for your answers.

1. At which if the three times A, B, D is the car traveling the fastest?

2. At which if the three times A, B, D is the car traveling the slowest?

3. What is the speed of the car at time B?

4. At which of the 3 times C, E, F is the car accelerating?

5. At which of the 3 times C, E, F is the car decelerating?

Q5]... [10 points] Is the function

$$f(x) = \begin{cases} x^2 + 3 & \text{if} & x \geq 0 \\ 3x + 2 & \text{if} & x < 0 \end{cases}$$

continuous at $x = 0$? Justify your answer.

Is the function

$$g(x) = \begin{cases} x \sin(\frac{1}{x}) & \text{if} & x \neq 0 \\ 0 & \text{if} & x = 0 \end{cases}$$

continuous at $x = 0$? Justify your answer.

Q6]...[8 points] Compute the following limit. Show details of your work.

$$\lim_{x \to 1} \frac{\sqrt{x} - 1}{\sqrt[3]{x} - 1}$$