Friday 10/21/2005 Midterm II 10:30am–11:20am

Name: Student ID:

Instructions.

1. Attempt all questions.
2. Do not write on back of exam sheets. Extra paper is available if you need it.
3. Show all the steps of your work clearly.

Question	Points	Your Score
Q1	10	
Q2	10	
Q3	10	
Q4	10	
Q5	10	
TOTAL	50	

Q1]. . . [10 points] Prove that the following are true for sets $A$and B.

$$(A \cup B) \cap \overline{(A \cap B)} \;=\; (A \cap \overline{B}) \cup (B \cap \overline{A})$$

$$A \cup (B \setminus A) \;=\; A \cup B$$

Q2]. . . [10 points] Suppose that $f : X \to Y$ is a function, and that $A \subset X$ and $B \subset X$.
Prove that $f(A \cap B) \subset f(A) \cap f(B)$.

Give an example to show that $f(A \cap B)$ need not be equal to $f(A) \cap f(B)$.

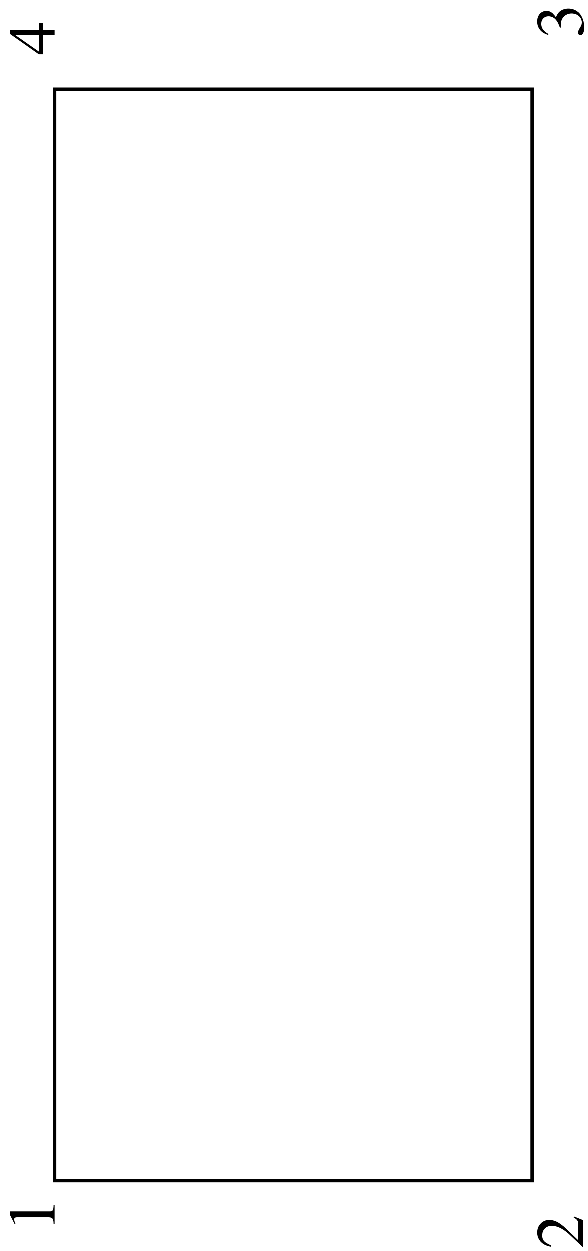
Prove that $f(A \cap B) = f(A) \cap f(B)$ under the additional assumption that f is an injective map.

Q3]. . . [10 points] For each of the following pairs of sets, say if they have the same cardinality or not. Give arguments (proofs) to justify your answers in each case.

$\mathbb{Z}^+$ and $\mathbb{Z}^+ \times \mathbb{Z}^+$.

$\mathbb{Z}^+$ and $(0, 1) = \{x \in \mathbb{R} | 0 < x < 1\}$.

Q4]... [10 points] How many symmetries does the rectangle below have? Describe them, and write down a composition table for them. Also, identify each symmetry with an element of the set of permutations Perm($\{1, 2, 3, 4\}$).



Q5]... [10 points] True/False. Give *reasons* for your answers. In these questions, capital letters A, B, C, X, Y denotes sets, and small letters are used to denote either functions (f, g) or elements of sets, y.

1. If $|A| = 3$ and $|B| = 4$, then $|A \cup B|$ must be equal to 7.

2. If $A \cup C = B \cup C$, then A must equal B.

3. If $A \oplus B = B$, then A must be $\emptyset$.

4. If $f \circ g$ is injective, then f must be injective.

5. If $f \circ g$ is surjective, then f must be surjective.

6. If $f : X \to Y$ is injective and $y \in Y$, then $|f^{-1}(\{y\})|$ must be 1.

7. The product of permutations (1234)(234) is equal to (1243).

8. The union of two disjoint countably infinite sets, is again countably infinite.

9. The composition of reflections in two perpendicular lines in the plane is equal to a 90° rotation about their intersection point.

10. If $|A| = 3$, $|B| = |C| = 5$, $|A \cap B| = 2$, $|B \cap C| = 3$, and $|A \cap C| = |A \cap B \cap C| = 1$, then $|A \cup B \cup C| = 8$.