SYMMETRIC DIFFERENCE IS ASSOCIATIVE

$$\boxed{S \oplus T = (S \cap \overline{T}) \cup (T \cap \overline{S})} \quad \text{——— (a)}$$

$$\boxed{\begin{aligned} S \oplus T &= (S \cup T) \cap \overline{(S \cap T)} \\ &= (S \cup T) \cap (\overline{S} \cup \overline{T}) \end{aligned}} \quad \text{——— (b)}$$

$$A \oplus (B \oplus C) = \left(A \cap \overline{(B \oplus C)}\right) \cup \left((B \oplus C) \cap \overline{A}\right) \quad \text{...... by (a)}$$

$$= \left[A \cap \overline{\left((B \cup C) \cap (\overline{B} \cup \overline{C})\right)}\right] \cup \left[\left((B \cap \overline{C}) \cup (C \cap \overline{B})\right) \cap \overline{A}\right] \quad \text{--- by (b) and (a)}$$

$$= \left[A \cap \left((\overline{B} \cap \overline{C}) \cup (B \cap C)\right)\right] \cup \left[\overline{A} \cap \left((B \cap \overline{C}) \cup (\overline{B} \cap C)\right)\right] \quad \text{---}$$

-- by De Morgan & commutativity of $\cap$

$$= \left(A \cap (\overline{B} \cap \overline{C})\right) \cup \left(A \cap (B \cap C)\right) \cup \left(\overline{A} \cap (B \cap \overline{C})\right) \cup \left(\overline{A} \cap (\overline{B} \cap C)\right)$$

--- by distrib. of $\cap$ over $\cup$.

——————— (*)

$$(A \oplus B) \oplus C = \left((A \oplus B) \cap \overline{C}\right) \cup \left(C \cap \overline{(A \oplus B)}\right) \quad \text{—— by (a)}$$

$$= \left[\left((A \cap \overline{B}) \cup (B \cap \overline{A})\right) \cap \overline{C}\right] \cup \left[C \cap \overline{\left((A \cup B) \cap (\overline{A} \cup \overline{B})\right)}\right] \quad \text{--- by (a) and (b)}$$

$$= \left[\left((A \cap \bar{B}) \cup (\bar{A} \cap B) \right) \cap \bar{C} \right] \cup$$

$$\left[\left((\bar{A} \cap \bar{B}) \cup (A \cap B) \right) \cap C \right]$$ --- by de Morgan & commut. of $\cap$.

$$= \left((A \cap \bar{B}) \cap \bar{C} \right) \cup \left((\bar{A} \cap B) \cap \bar{C} \right) \cup$$

$$\left((\bar{A} \cap \bar{B}) \cap C \right) \cup \left((A \cap B) \cap C \right)$$ --- by distrib of $\cap$ over $\cup$

——— $(**)$

Finally, $(*) = (**)$ by associativity of $\cap$ & comm/assoc of $\cup$.

Thus $(A \oplus B) \oplus C = A \oplus (B \oplus C)$

SYMM DIFF is ASSOCIATIVE.