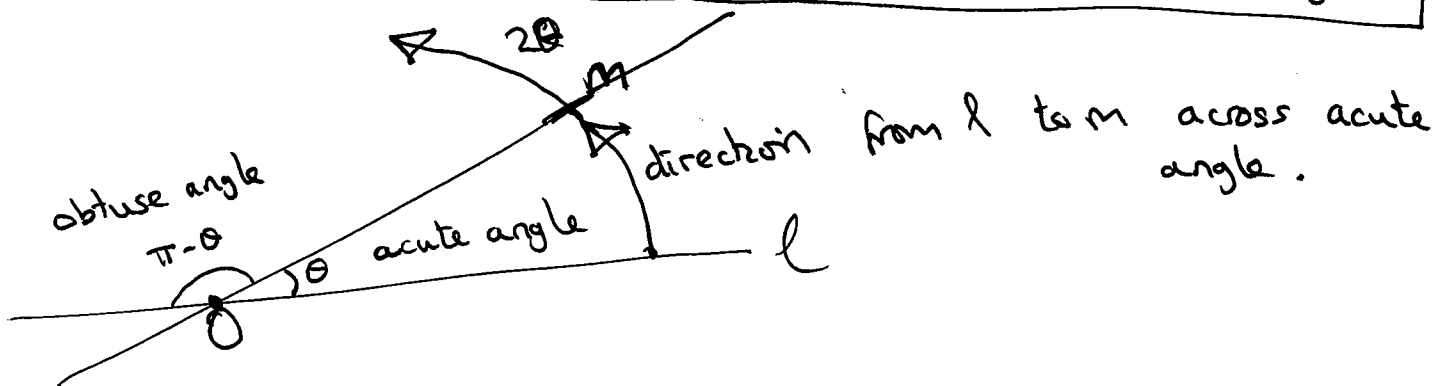


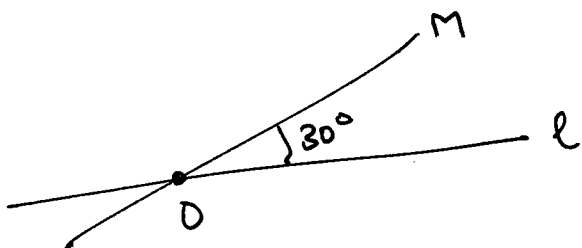
Cool Fact : Let l denote 'reflection in the line l '
and m denote 'reflection on the line m '.

If lines l, m meet in a point O , and
make an angle of θ , then:

$m \circ l =$ Rotation about O , through an
angle of 2θ , in direction from
 l to m (across acute angle).



eg



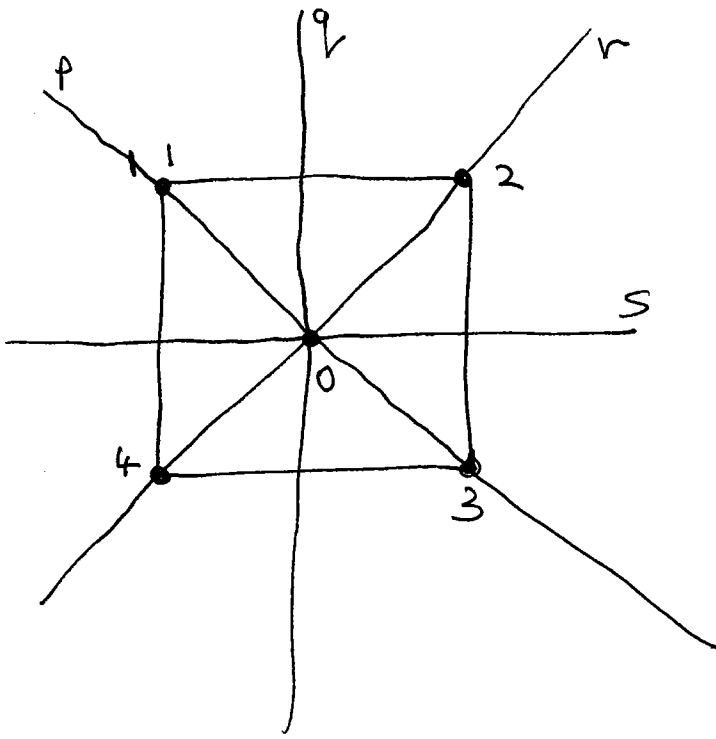
$m \circ l =$ counterclockwise
rotation about O
through $2(30) = 60^\circ$.

$l \circ m =$ clockwise rotation
about O through
 $2(30) = 60^\circ$.

Note:

$$(l \circ m) \circ (m \circ l) = l \circ m^2 \circ l = \mathbb{1}$$

$= \mathbb{1}$ makes sense,
since clockwise
+ counterclockwise rotations
cancel out!



$$p \leftrightarrow (24)$$

$$q \leftrightarrow (12)(34)$$

$$r \leftrightarrow (13)$$

$$s \leftrightarrow (14)(23)$$

$$R \leftrightarrow (1432)$$

$$R^2 \leftrightarrow (13)(24)$$

$$R^3 \leftrightarrow (1234)$$

$$\mathbb{1} = R^4 \leftrightarrow \mathbb{1}$$

R = counter clock wise rotation about 0 through $\pi/2$.

$$R^2 = R \circ R$$

$$R^3 = R \circ R \circ R$$

$$R^4 = R \circ R \circ R \circ R = \mathbb{1}$$

compositions
symbol \rightarrow

	$\mathbb{1}$	R	R^2	R^3	p	q	r	s
$\mathbb{1}$	$\mathbb{1}$	R	R^2	R^3	p	q	r	s
R	R	R^2	R^3	$\mathbb{1}$	s	p	q	r
R^2	R^2	R^3	$\mathbb{1}$	R	r	s	p	q
R^3	R^3	$\mathbb{1}$	R	R^2	q	r	s	p
p	p	q	r	s	$\mathbb{1}$	R	R^2	R^3
q	q	r	s	p	R^3	$\mathbb{1}$	R	R^2
r	r	s	p	q	R^2	R^3	$\mathbb{1}$	R
s	s	p	q	r	R	R^2	R^3	$\mathbb{1}$

IDEA

- TOP LEFT SQUARE IS EASY!!
- Bottom RIGHT SQUARE is OK!!
(Just use "cool fact" over & over again.)
- For remaining squares use algebra!

eg

$$R \circ P = (\underbrace{S \circ P}) \circ P = S \circ (P \circ P) \\ = S \circ \mathbb{1} \\ = S$$

Read ~~P~~ from bottom right square!

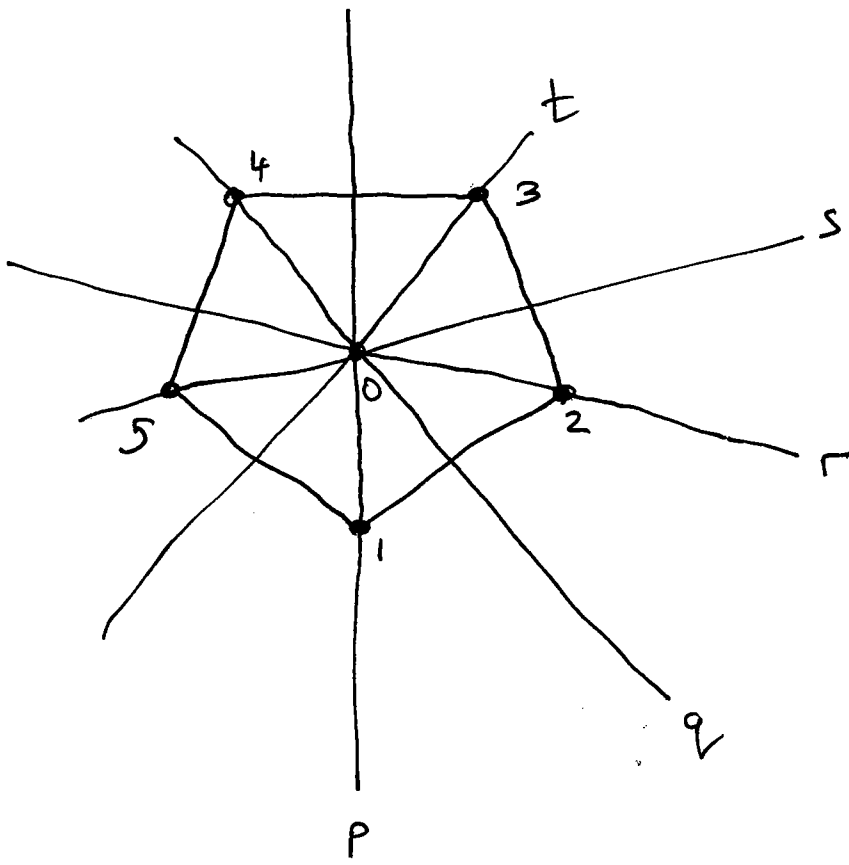
and eg

$$P \circ R = P \circ (\underbrace{P \circ Q}) = (P \circ P) \circ Q \\ = \mathbb{1} \circ Q \\ = Q$$

Read from bottom right square!

There are very fast ways of reading off & filling in values — via Rows/columns of original table.

Think about it!!



$$p \leftrightarrow (25)(43)$$

$$q \leftrightarrow (12)(35)$$

$$r \leftrightarrow (45)(13)$$

$$s \leftrightarrow (14)(23)$$

$$t \leftrightarrow (15)(24)$$

$$R \leftrightarrow (12345)$$

$$R^2 \leftrightarrow (13524)$$

$$R^3 \leftrightarrow (14253)$$

$$R^4 \leftrightarrow (15432)$$

$$I = R^5 \leftrightarrow I$$

$R =$ rotation counter clockwise about O through $\frac{2\pi}{5}$

$$R^2 = R \circ R, \quad R^3 = R \circ R^2, \quad R^4 = R^3 \circ R, \quad R^5 = R^4 \circ R = I.$$

There are 5 reflections in lines p, q, r, s, t and 5 rotations about O : $R, R^2, R^3, R^4, R^5 = I$.

Comp table.

o	$\mathbb{1}$	R	R^2	R^3	R^4	p	q	r	s	t
$\mathbb{1}$	$\mathbb{1}$	R	R^2	R^3	R^4	p	q	r	s	t
R	R	R^2	R^3	R^4	$\mathbb{1}$	q	r	s	t	p
R^2	R^2	R^3	R^4	$\mathbb{1}$	R	r	s	t	p	q
R^3	R^3	R^4	$\mathbb{1}$	R	R^2	s	t	p	q	r
R^4	R^4	$\mathbb{1}$	R	R^2	R^3	t	p	q	r	s
p	p	t	s	r	q	$\mathbb{1}$	R^4	R^3	R^2	R
q	q	p	t	s	r	R	$\mathbb{1}$	R^4	R^3	R^2
r	r	q	p	t	s	R^2	R	$\mathbb{1}$	R^4	R^3
s	s	r	q	p	t	R^3	R^2	R	$\mathbb{1}$	R^4
t	t	s	r	q	p	R^4	R^3	R^2	R	$\mathbb{1}$

Some patterns should emerge as you fill in the squares of $\mathbb{1}$

