Math 2433-001 Quiz 5 Name: SOLUTIONS

Q1].. State the integral test for series.

Given $\sum_{n=1}^{\infty} a_n$ where $a_n = f(x)$ and $\left\{ \begin{array}{l} \cdot\ f(x) > 0 \\ \cdot\ f(x) \text{ continuous} \\ \cdot\ f(x) \downarrow 0 \end{array} \right\}$ on some interval $[b, \infty)$.

Then $\sum_{n=1}^{\infty} a_n$ is convgt/divgt $\Longleftrightarrow \int_b^{\infty} f(x)\,dx$ is convgt/divgt.

Verify that

$$\sum_{n=3}^{\infty} \frac{\ln n}{n}$$

satisfies the conditions of the integral test.

for $[3,\infty)$ we have $\ln(x) > 0 \Rightarrow \frac{\ln(x)}{x} > 0$ positive

$\frac{\ln(x)}{x}$ is continuous on $[3,\infty)$ since $\ln(x)$ & x are & $x \neq 0$.

$$\frac{d}{dx}\left(\frac{\ln(x)}{x}\right) = \frac{x\left(\frac{1}{x}\right) - \ln(x)\cdot 1}{x^2} = \frac{1-\ln(x)}{x^2} < 0 \text{ for } x > e$$

$\ln(x) > \ln(e) = 1$

$\Rightarrow \frac{d}{dx}\left(\frac{\ln(x)}{x}\right) < 0$ on $[3,\infty)$

$\Rightarrow \frac{\ln(x)}{x} \downarrow$ on $[3,\infty)$.

Use the integral test to see whether the series above converges or diverges.

$$\int_3^{\infty} \frac{\ln(x)}{x}\,dx = \lim_{t\to\infty} \int_3^{t} \frac{\ln(x)}{x}\,dx$$

$u = \ln(x)$, $du = \frac{dx}{x}$

$$= \lim_{t\to\infty} \int_{\ln(3)}^{\ln(t)} u\,du$$

$$= \lim_{t\to\infty} \left.\frac{u^2}{2}\right]_{\ln(3)}^{\ln(t)} = \lim_{t\to\infty}\left(\frac{\ln(t)^2}{2} - \frac{\ln(3)^2}{2}\right) = \infty$$

$\int$ divgt $\Rightarrow \sum$ divgt by Integral Test