Q1]... [15 points] Let $f(x)$ be a function of x whose domain is all real numbers. Consider the following expression.

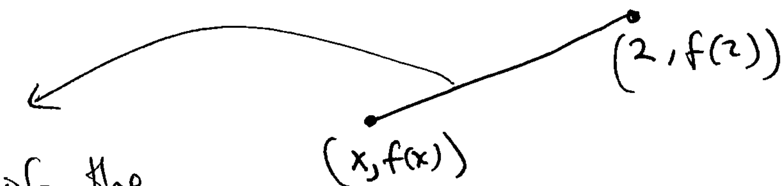
$$\frac{f(x) - f(2)}{x - 2}$$

What is this expression called?

THIS IS CALLED A <u>DIFFERENCE QUOTIENT</u>.

Write down two interpretations of this expression.

① <u>Geometrical.</u>



It is the slope of the line segment connecting two points on the graph of $f(x)$; namely, $(x, f(x))$ and $(2, f(2))$

② <u>Analytical.</u>

It is the average rate of change of $f(x)$ with respect to x over the interval $[x, 2]$ (OR $[2, x]$ if $2 < x$).

$\leftarrow$ if $x < 2$.

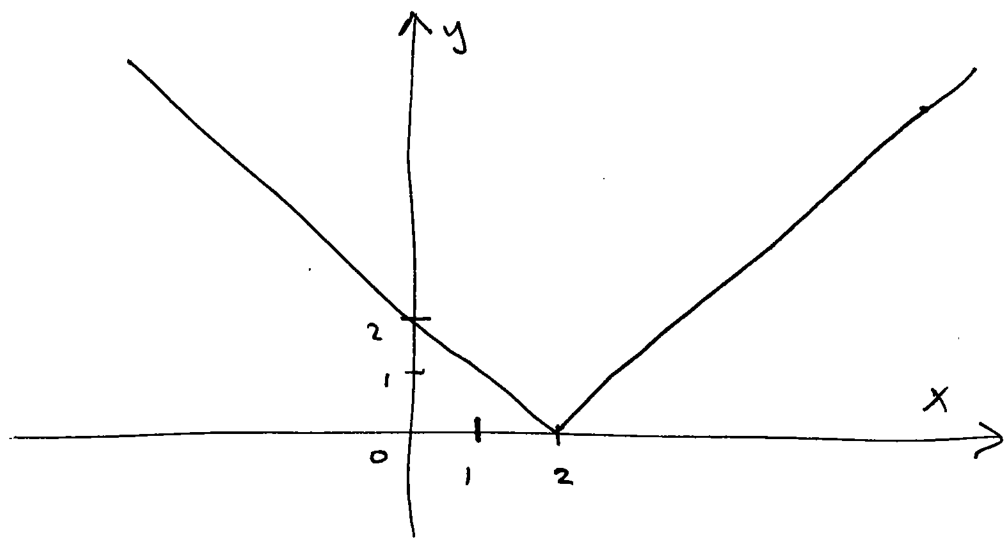
$$\frac{f(x) - f(2)}{x - 2} \quad \begin{array}{l} \leftarrow \text{change in output values } (f(x)\text{-values}) \\ \leftarrow \text{change in (corresponding) input values } (x\text{-values}) \end{array}$$

Q2]... [20 points] For each of the two functions below, write down the **domain**, the **range** and **sketch the graph** of the function.

$$f(x) = |x - 2|$$

Domain $(-\infty, \infty)$

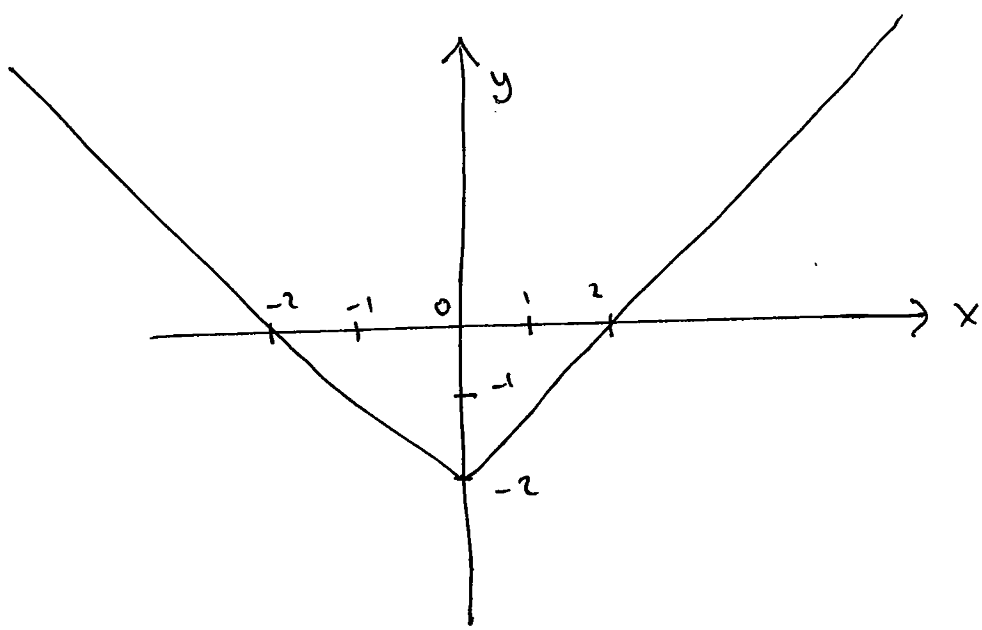
Range $[0, \infty)$



$$g(x) = |x| - 2$$

Domain $= (-\infty, \infty)$

Range $= [-2, \infty)$



Q3]... **[20 points]** Compute the following limit. Show all the steps of your work.

$$\lim_{x\to 9} \frac{\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{9}}}{x-9}$$

$$= \lim_{x\to 9} \frac{\frac{\sqrt{9}}{\sqrt{9}\sqrt{x}} - \frac{\sqrt{x}}{\sqrt{9}\sqrt{x}}}{(x-9)} = \lim_{x\to 9} \left(\frac{\sqrt{9}-\sqrt{x}}{\sqrt{9}\sqrt{x}}\right) \cdot \left(\frac{1}{x-9}\right)$$

write as

$$(\sqrt{x})^2 - (\sqrt{9})^2$$
$$= (\sqrt{x}-\sqrt{9})(\sqrt{x}+\sqrt{9})$$

$$= \lim_{x\to 9} \left(\frac{\overset{-1}{\cancel{(\sqrt{9}-\sqrt{x})}}}{\sqrt{9}\sqrt{x}} \frac{1}{\underset{1}{\cancel{(\sqrt{x}-\sqrt{9})}}(\sqrt{x}+\sqrt{9})} \right)$$

$$= \lim_{x\to 9} \left(\frac{-1}{\sqrt{9}\sqrt{x}\,(\sqrt{9}+\sqrt{x})} \right)$$

$$= \frac{-1}{\sqrt{9}\sqrt{9}\,(\sqrt{9}+\sqrt{9})} = \frac{-1}{9(3+3)} = \boxed{\frac{-1}{54}}$$

Q4]...**[20 points]** Compute the following limit. Show all the steps of your work.

$$\lim_{h\to 0} \frac{(2+h)^3 - 8}{h}$$

1

1 1

1 2 1

1 3 3 1

coefficients in cube expansion....

$$= \lim_{h\to 0} \left[\frac{1.(2)^3 + 3.(2)^2(h) + 3.(2)(h)^2 + 1.(h)^3 - 8}{h} \right]$$

$$= \lim_{h\to 0} \left(\frac{\cancel{8} + 12h + 6h^2 + h^3 - \cancel{8}}{h} \right)$$

$$= \lim_{h\to 0} \left(\frac{\cancel{h}(12 + 6h + h^2)}{\cancel{h}} \right)$$

cancel those since $h \neq 0$ when taking $\lim_{h\to 0}$

$$= \lim_{h\to 0} (12 + 6h + h^2)$$

$$= 12 + 6(0) + (0)^2$$

$$= \boxed{12}$$

Q5]... **[15 points]** Find the value of c which makes the following function continuous. Show the details of your reasoning. Keep in mind that the steps of your argument (how you arrived at a value for c) are more important than the actual value of c.

$$f(x) = \begin{cases} 2x+4 & \text{for } x \le 1 \\ cx^2 + c & \text{for } x > 1 \end{cases}$$

$f(1) = 2(1)+4 = 2+4 = 6.$

If $f(x)$ is to be continuous, then

$\lim_{x \to 1} f(x)$ should exist & be equal to $f(1) = 6$.

In particular, $\lim_{x \to 1^+} f(x) = 6$

$x > 1 \Rightarrow$ use cx^2+c portion of the def^n of $f(x)$

$\Rightarrow \lim_{x \to 1^+} (cx^2+c) = 6$

$\Rightarrow c(1)^2 + c = 6$

$\Rightarrow c + c = 6$

$\Rightarrow 2c = 6$

$\Rightarrow \boxed{c = 3} \leftarrow$

Q6]...[10 points] Suppose that $\sin(\theta) = 3/5$ and that $\pi/2 < \theta < \pi$. Find the values of $\cos(\theta)$ and of $\tan(\theta)$. Show the details of your work.

$$\cos^2(\theta) + \sin^2(\theta) = 1 \quad \leftarrow$$

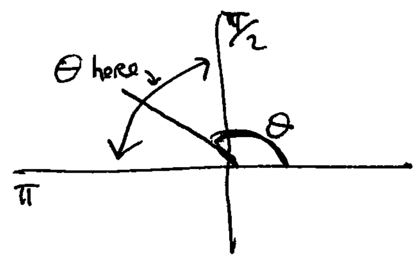
BASIC PROPERTY (UNIT CIRCLE COORDINATES)

$$\cos^2(\theta) = 1 - \sin^2(\theta)$$

$$\cos(\theta) = \pm\sqrt{1 - \sin^2(\theta)}$$

.... Which sign do we use?

Now $\pi/2 < \theta < \pi$



$$\Rightarrow \underline{\cos(\theta) < 0}$$

$$\Rightarrow \cos(\theta) = -\sqrt{1 - \sin^2(\theta)}$$

In our example, $\sin(\theta) = 3/5$. Therefore...

$$\Rightarrow \cos(\theta) = -\sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$= -\sqrt{\frac{25 - 9}{25}} = -\sqrt{\frac{16}{25}} = -\frac{4}{5}$$

$$\boxed{\cos(\theta) = \frac{-4}{5}}$$

$$\& \quad \tan(\theta) = \frac{\sin(\theta)}{\cos\theta} = \frac{\frac{3}{5}}{-\frac{4}{5}} = \left(\frac{3}{5}\right)\left(-\frac{5}{4}\right)$$

$$= -\frac{3}{4}$$

$$\boxed{\tan\theta = -\frac{3}{4}}$$