Friday 03/12/2004   Midterm II   8:30am–9:20am

Name: Student ID:

Instructions.

1. No calculators or notes.

2. Attempt all questions.

3. Do not write on back of exam sheets. Extra paper is available.

4. Show all the steps of your work clearly.
   The method (reasoning) used to obtain an answer is worth more than the answer itself.

Question	Points	Your Score
Q1	12	
Q2	10	
Q3	12	
Q4	10	
Q5	11	
TOTAL	55	

Q1]...[12 points] Compute the derivative $f'(x)$ of each of the following functions. Show details of your work.

$$f(x) = \sqrt{\sin(x^2)}$$

$$f(x) = (3x - 2)^{50}(x + 3)^{40}$$

$$f(x) = \frac{5x^3 + 2x + 3}{x^2 + 1}$$

Q2]... [10 points] Write down the limit definition of the derivative (the $h \to 0$ limit form).

Write down the product rule (formula is sufficient).

Give a proof of the product rule.

Q3]...[12 points] Use implicit differentiation to find y' for the ellipse

$$x^2 + 2y^2 \;=\; 2$$

Use implicit differentiation to find y' for the hyperbola

$$2x^2 - 2y^2 = 1$$

Show that the ellipse and the hyperbola above intersect orthogonally (meet at right angles).

Q4]...[10 points] Find the equation of the normal line to the graph of $y = x^2$ at the point $(1,1)$. [Key fact: Normal line is perpendicular to tangent line at point of contact]

Verify that $(3,0)$ lies on this normal line.

Which point on the parabola $y = x^2$ is closest to the point $(3,0)$? Justify your answer (say why your point is closer than all other points on the parabola).

Q5]...[11 points] Evaluate the following limit by first recognizing it as a derivative of some function.

$$\lim_{h \to 0} \frac{\tan(\frac{\pi}{4} + h) - 1}{h}$$

Find an expression for the n-th derivative $f^{(n)}(x)$ of the function

$$f(x) = \frac{1}{1 - x}$$