Properties of "divides". Below $a, b, c \in \mathbb{Z}$, $a > 0$.

1. If $a \mid b$ and $p \in \mathbb{Z}$, then $a \mid pb$.

<u>Proof:</u> $a \mid b \Rightarrow b = aq$ for some $q \in \mathbb{Z}$

Then $pb = p(aq) = a(\underbrace{pq}_{\text{in } \mathbb{Z}})$ is a multiple of a

$\Rightarrow a \mid pb$ ▨

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2. If $a \mid b$ and $a \mid c$, then $a \mid (b+c)$.

<u>Proof:</u> $a \mid b \Rightarrow b = ak$ some $k \in \mathbb{Z}$

$a \mid c \Rightarrow c = al$ some $l \in \mathbb{Z}$

$\Rightarrow b + c = ak + al = a(\underbrace{k+l}_{\text{this is in } \mathbb{Z}})$

$\Rightarrow a \mid (b+c)$ ▨

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3. $\boxed{\text{Corollary of 1 \& 2}}$ If $a \mid b$ and $a \mid c$ and $p, q \in \mathbb{Z}$, then $a \mid (pb + qc)$.

<u>Proof:</u> $a \mid b \Rightarrow a \mid pb$ by 1

$a \mid c \Rightarrow a \mid qc$ by 1

$a \mid pb$ and $a \mid qc \Rightarrow a \mid (pb + qc)$ by 2 ▨
