For all $n \in \mathbb{N}$, $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

Proof We argue by induction on n. Let $P(n)$ be the statement

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

P(1) is true: $1^2 = 1 \overset{??}{=} \frac{1(1+1)(2(1)+1)}{6} = \frac{1(2)(3)}{6} = \frac{6}{6} = 1$ yes!

P(k) true $\longrightarrow$ P(k+1) true! Assume $P(k)$ is true. That is

$$1^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$$

Add $(k+1)^2$ to both sides to get

$$1^2 + \cdots + k^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{(k+1)\,k\,(2k+1)}{6} + \frac{(k+1)^2\,6}{6}$$

$$= \frac{(k+1)\left[k(2k+1) + 6(k+1)\right]}{6}$$

$$= \frac{(k+1)(2k^2 + 7k + 6)}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

$$= \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$$

That shows P(k+1) is true!

By P. of I., $P(n)$ is true $\forall n \in \mathbb{N}$.

$$\forall n \in \mathbb{N}, \quad \frac{d^n}{dx^n}(e^{4x}) = 4^n e^{4x}$$

<u>Proof</u> We use induction on n.

Let $P(n)$ be the statement $\frac{d^n}{dx^n}(e^{4x}) = 4^n e^{4x}$.

<u>$P(1)$ is true:</u> $\frac{d^1}{dx^1}(e^{4x}) = \frac{d}{dx}(e^{4x}) = e^{4x} \cdot \frac{d(4x)}{dx}$ --- Ch. Rule (calc I)

$$= 4e^{4x} = 4^1 \cdot e^{4x}$$

<u>$P(k)$ true $\longrightarrow P(k+1)$ true:</u> Assume hypothesis of this conditional statement.

ie. Assume $P(k)$ is true.

ie. $\frac{d^k}{dx^k}(e^{4x}) = 4^k \cdot e^{4x}$

Now take $\frac{d}{dx}$ of both sides of this equation to get

$$\frac{d}{dx}\left(\frac{d^k}{dx^k}(e^{4x})\right) = \frac{d}{dx}(4^k \cdot e^{4x})$$

$$= 4^k \frac{d}{dx}(e^{4x})$$

This is $\frac{d^{k+1}}{dx^{k+1}}(e^{4x})$

$$= 4^k(4e^{4x})$$

$$= 4^{k+1} \cdot e^{4x}$$ ie. $P(k+1)$ is true.

Therefore, $P(n)$ is true $\forall n \in \mathbb{N}$ by the P. of I.

$\forall n \in \mathbb{N}, \quad 1 + 5 + 9 + \cdots + (4n-3) = n(2n-1)$

<u>Proof</u> By induction on n. Let $P(n)$ be the statement

$$1 + 5 + \cdots + (4n-3) = n(2n-1).$$

<u>$P(1)$ is true:</u> $1 \stackrel{?}{=} 1(2(1)-1) = 1(2-1) = 1(1) = 1$ yes!

<u>$P(k)$ true $\rightarrow$ $P(k+1)$ true:</u> Assume $P(k)$ is true. That is

$$1 + 5 + \cdots + (4k-3) = k(2k-1)$$

Add $(4(k+1)-3)$ to both sides of this equation

$$\begin{aligned} 1 + 5 + \cdots + (4k-3) + (4(k+1)-3) &= k(2k-1) + (4(k+1)-3) \\ &= 2k^2 - k + 4k + 4 - 3 \\ &= 2k^2 + 3k + 1 \\ &= (2k+1)(k+1) \\ &= (k+1)(2(k+1)-1) \end{aligned}$$

Thus $P(k+1)$ is true.

By the P. of I., $P(n)$ is true $\forall n \in \mathbb{N}$.

For all $n \in \mathbb{N}$ $\quad 1+2+\dots+n = \frac{n(n+1)}{2}$

<u>Proof</u> We argue by induction on n. Let $P(n)$ denote the statement $1+2+\dots+n = \frac{n(n+1)}{2}$.

<u>$P(1)$ is true</u>: $1 \overset{?}{=} \frac{1(1+1)}{2} = \frac{1(2)}{2} = \frac{2}{2} = 1$ yes!

<u>$P(k)$ true $\longrightarrow$ $P(k+1)$ true</u>: Assume $P(k)$ is true. That is

$$1 + \dots + k = \frac{k(k+1)}{2}$$

Adding $(k+1)$ to both sides of this equation gives

$$\begin{aligned} 1+\dots+k+(k+1) &= \frac{k(k+1)}{2} + (k+1) \\ &= \frac{k(k+1)}{2} + \frac{2(k+1)}{2} \\ &= \frac{(k+2)(k+1)}{2} \\ &= \frac{(k+1)((k+1)+1)}{2} \end{aligned}$$

Thus $P(k+1)$ is true.

By P. of I., $P(n)$ is true $\forall n \in \mathbb{N}$.