

Friday 09/18/2015

Midterm I

50 minutes

Name: Student ID: **Instructions.**

1. Attempt all questions.
2. Do not write on back of exam sheets. Extra paper is available if you need it.
3. Show all the steps of your work clearly.

Question	Points	Your Score
Q1	25	
Q2	25	
Q3	25	
Q4	25	
TOTAL	100	

Miscellaneous expressions and definitions.

1.

$$\ln(x) = \int_1^x \frac{dt}{t}$$

2.

e^x is the inverse of $\ln(x)$

3.

$$\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

4.

$$\frac{d}{dx} \cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$$

5.

$$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$$

6.

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

7.

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

8.

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

9.

$$\cosh^2(x) - \sinh^2(x) = 1$$

Q1]... [25 points]

1. Compute the derivative of the function $y = (\sin x)^x - x^{\sin x}$

2. Compute the derivative of the function $y = \log_{\pi}(x)$.

Q2]. . . [25 points]

1. Using the definitions of the hyperbolic trigonometric functions, verify that

$$\frac{d \cosh x}{dx} = \sinh x$$

2. Verify that the function

$$y = \frac{T_0}{k} \cosh \left(\frac{k}{T_0} x \right)$$

is a solution of the freely hanging cable equation

$$\frac{d^2 y}{dx^2} = \frac{k}{T_0} \sqrt{1 + \left(\frac{dy}{dx} \right)^2}$$

Here k and T_0 are constants which depend on the cable.

Q3]... [25 points] A radioactive material has a half-life of 1,200 years. Your answers to the questions below will be numbers; it is OK to describe these numbers as expressions involving other numbers. Since you do not have a calculator, I am **not** expecting you to give answers as explicit numbers with many decimal places accuracy.

1. What percentage of the original sample is left after 20 years?

2. How long does it take for 20% of the original sample to decay?

Q4]... [25 points] Compute the following antiderivatives.

1.

$$\int \frac{dx}{x \ln x}$$

2.

$$\int \frac{dx}{7 + (x - 1)^2}$$