1. **Log and Exp.** $\ln(x) = \int_1^x \frac{dt}{t}$ $\quad$ e^x is the inverse of $\ln(x)$

2. **Inverse trig.**
$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$ $\quad$ $\frac{d}{dx}\cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$ $\quad$ $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$ $\quad$ $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}(\frac{x}{a})$

3. **Hyperbolic trig.**
$\cosh(x) = \frac{e^x+e^{-x}}{2}$ $\quad$ $\sinh(x) = \frac{e^x-e^{-x}}{2}$ $\quad$ $\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$ $\quad$ $\cosh^2(x) - \sinh^2(x) = 1$

4. **Trig Addition, Half Angle.**
$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$
$\cos(2A) = \cos^2(A) - \sin^2(A)$ $\quad$ $\cos(2A) = 2\cos^2(A) - 1$ $\quad$ $\cos(2A) = 1 - 2\sin^2(A)$
$\sin^2(x) = (1-\cos(2x))/2$ $\quad$ $\cos^2(x) = (1+\cos(2x))/2$
$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$ $\quad$ $\sin(2x) = 2\sin(x)\cos(x)$.

5. **Integration by Parts.**
$\int u\,dv = uv - \int v\,du$

6. **Trig Substitutions.**
For $\sqrt{a^2-x^2}$ use $x = a\sin(\theta)$ $\quad$ For $\sqrt{a^2+x^2}$ use $x = a\tan(\theta)$ $\quad$ For $\sqrt{x^2-a^2}$ use $x = a\sec(\theta)$

7. **Some integrals.**

$$\int \tan(x)\,dx = \ln|\sec(x)| + C \qquad \int \sec(x)\,dx = \ln|\sec(x)+\tan(x)| + C$$

8. **Geometric Series.**
$\sum_{n=1}^{\infty} ar^{n-1}$ converges when $|r| < 1$; it converges to the sum $\frac{a}{1-r}$ when $|r| < 1$.

9. **Test for Divergence.**
If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

10. **Integral Test.**
For $f(x)$ continuous on $[1,\infty)$, positive and decreasing to 0, the series $\sum_{n=1}^{\infty} f(n)$ converges if and only if the improper integral $\int_1^{\infty} f(x)dx$ converges.

11. **Comparison Tests.**
Direct comparison test: compares series of positive terms, term-by-term.

Limit comparison test: compares series of positive terms $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ when $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ a finite limit not equal to 0.

12. **Root Test.**
Let $\lim_{n\to\infty} |a_n|^{1/n} = L$. If $L < 1$ then $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, and if $L > 1$ then it is divergent.

13. **Ratio Test.**
Let $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = L$. If $L < 1$ then $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, and if $L > 1$ then it is divergent.

14. **Alternating Series Test.**
If a_n are positive, decreasing to 0, then $\sum_{n=1}^{\infty}(-1)^{n-1}a_n$ is convergent. Moreover, the nth partial sum is within a_{n+1} of the sum of the whole series.

15. **Power series.**
Ratio test is useful for computing the radius of convergence of a power series $\sum_{n=0}^{\infty} c_n(x-a)^n$.

16. **Taylor and Maclaurin Series.**
Taylor series for $f(x)$ centered about a is given by

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Maclaurin series for $f(x)$ is the Taylor series for $f(x)$ centered about 0.

17. **Remainder Estimate.**
Taylor's inequality states that if $|f^{(n+1)}(x)| \leq M$ on the interval $[a-d, a+d]$, then

$$|f(x) - T_n(x)| \;\leq\; \frac{M|x-a|^{(n+1)}}{(n+1)!}$$

on the interval $[a-d, a+d]$. Here $T_n(x)$ is the *degree* n *Taylor polynomial approximation* to $f(x)$.

18. **Parametric curves.**
Arc length $= \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$ Slope $\frac{dy}{dx} = \frac{y'(t)}{x'(t)}$

19. **Polar curves.** $x = r\cos\theta$ $y = r\sin\theta$ $x^2 + y^2 = r^2$ $\tan\theta = y/x$

Arc length $= \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2}\, d\theta$ Polar area $= \int_a^b \frac{r^2}{2}\, d\theta$

20. **Dot product of vectors.**
$\langle u_1, u_2\rangle \cdot \langle v_1, v_2\rangle = u_1v_1 + u_2v_2$ $\langle u_1, u_2, u_3\rangle \cdot \langle v_1, v_2, v_3\rangle = u_1v_1 + u_2v_2 + u_3v_3$

$\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos\theta$ $\mathrm{Comp}_{\mathbf{u}}(\mathbf{v}) = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}|}$ $\mathrm{Proj}_{\mathbf{u}}(\mathbf{v}) = \frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{u}\cdot\mathbf{u}}\mathbf{u}$

21. **Cross product of 3-dimensional vectors.**

$$\langle u_1, u_2, u_3\rangle \times \langle v_1, v_2, v_3\rangle \;=\; \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} \;=\; \mathbf{i}\begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} + \mathbf{j}\begin{vmatrix} u_3 & u_1 \\ v_3 & v_1 \end{vmatrix} + \mathbf{k}\begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$$

$|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}|\sin\theta$ gives the area of a parallelogram with edges $\mathbf{u}$ and $\mathbf{v}$.

$\mathbf{u} \times \mathbf{v}$ is perpendicular to $\mathbf{u}$ and to $\mathbf{v}$.

22. **Triple product.** Gives signed volume of parallelepiped with sides $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$.

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) \;=\; \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$