Calculus IV [2443–004] Midterm III

For full credit, give reasons for all your answers.

Q1]...[15 points] Evaluate the following triple integral by first sketching the region of integration, and then converting it to a spherical coordinates integral.

$$\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{0}\int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} z\,dzdxdy$$

Q2]...[20 points] Write down the equation in the statement of Green's theorem, indicating what the various parts of it stand for.
Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$ directly, where $\mathbf{F} = \langle -x^2y^2, xy\rangle$ and C is the positively oriented boundary of the region bounded by the y-axis, the line $y = 1$, and the curve $y = \sqrt{x}$.
Use Green's theorem to compute the path integral above by a second method. Compare your answers.

Q3]...[20 points] State the fundamental theorem for path integrals.
Let $\mathbf{F} = \langle\, ye^{yz}\cos(xy),\ ze^{yz}\sin(xy) + xe^{yz}\cos(xy),\ ye^{yz}\sin(xy)\,\rangle$. Show that $\mathbf{curl}(F) = \mathbf{0}$.
Find a function f so that $\mathbf{F} = \nabla f$.
Use the fundamental theorem to give a quick computation of the path integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ where C is the straight line curve from $(0, e, \pi/2)$ to $(\pi, 1/2, 0)$.

Q4]...[5 points] Determine (giving reasons) whether the following vector field has positive, negative or zero divergence.