We use the same notation as in question 1 of the discovery project. $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$ and $\mathbf{v_4}$ are the outward pointing normal vectors to the four faces of an arbitrary (not necessarily right angled) tetrahedron. The lengths of the $\mathbf{v_i}$ are equal to the areas of the corresponding faces of the tetrahedron.

1. Use the result of Q1 to prove the following generalization of the law of cosines

   $$|\mathbf{v_4}|^2 = |\mathbf{v_1}|^2 + |\mathbf{v_2}|^2 + |\mathbf{v_3}|^2 + 2|\mathbf{v_1}||\mathbf{v_2}|\cos\theta_{12} + 2|\mathbf{v_2}||\mathbf{v_3}|\cos\theta_{23} + 2|\mathbf{v_3}||\mathbf{v_1}|\cos\theta_{31}$$

   where θ_{ij} denotes the angle between the normal vectors $\mathbf{v_i}$ and $\mathbf{v_j}$.

2. Say why this is a generalization of the law of cosines. Remember that the law of cosines for a triangle with sides of length a, b and c states that

   $$c^2 = a^2 + b^2 - 2ab\cos C$$

   where C is the angle between the sides of length a and b. In particular, explain the difference in signs in the trig terms.