Math 2423–001 Calculus II Final **NAME:** _____

Attempt all questions. Total Points: 120.

Q1]. . . [10 points] Evaluate the following limit.

$$\lim_{x\to\infty}\left(1+\sin\left(\frac{3}{x}\right)\right)^{x}$$

Q2]. . . [10 points] Evaluate the trigonometric integral

$$\int \sin^4 x\, dx$$

Q3]. . . [10 points] Use a trigonometric substitution to compute the following indefinite integral

$$\int x^3\sqrt{9-x^2}\,dx$$

Q4]. . . [10 points] Use integration by parts to evaluate the following indefinite integral

$$\int \tan^{-1} x\,dx$$

Q5]. . . [10 points] Compute the arc length of the function $f(x) = \cosh x$ from $x = 0$ to $x = a$.

Q6]. . . [10 points] Compute the area of the region which lies below the graph of $\frac{1}{x \ln x}$, above the x-axis, and to the right of the line $x = e$.

Q7]. . . [10 points] Suppose that $f''(x)$ exists, and that f has an inverse function g. Suppose also that $g(4) = 5$, $g(5) = 4$, $f'(5) = 7$, $f''(5) = 3$, $f'(4) = 2$, and $f''(4) = 11$. Find $g''(4)$.

Q8]. . . [10 points] Evaluate the following limit

$$\lim_{n\to\infty} \left(n \int_3^{3+\frac{1}{n}} \sin(t^2)\, dt \right)$$

Q9]. . . [20 points] Find the coordinates of the center of mass (centroid) of the region in the plane which is between the graphs of $y = x$ and $y = x^2$.

Use Pappus theorem to compute the two volumes of revolution obtained by rotating the region above about (i) the x-axis, and about (ii) the line $x = -1$.

Write down integrals for the volumes in (i) and (ii) above. Use the washer method for the volume in (i), and the shell method for the volume in (ii). You do NOT have to evaluate these integrals.

Q10]. . . [20 points] Use the factorization $x^3 + 1 = (x + 1)(x^2 - x + 1)$ to help you evaluate the following integral. Show all your work.

$$\int \frac{dx}{x^3 + 1}$$