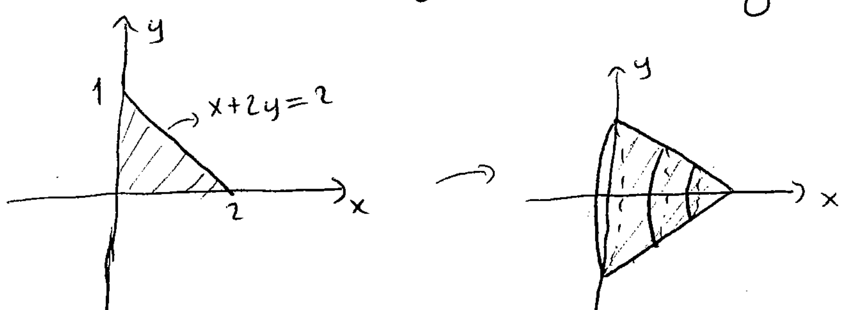
Hmk #4

Section 6.2

2) $x+2y=2 \iff y = 1 - \frac{1}{2}x$

hence $\text{Area} = A(x) = \pi\left(1-\frac{1}{2}x\right)^2$

$$V = \int_0^2 \pi y^2 dx = \pi \int_0^2 \left(1-\frac{1}{2}x\right)^2 dx = \pi \int_0^2 \left(1 - x + \frac{1}{4}x^2\right) dx$$



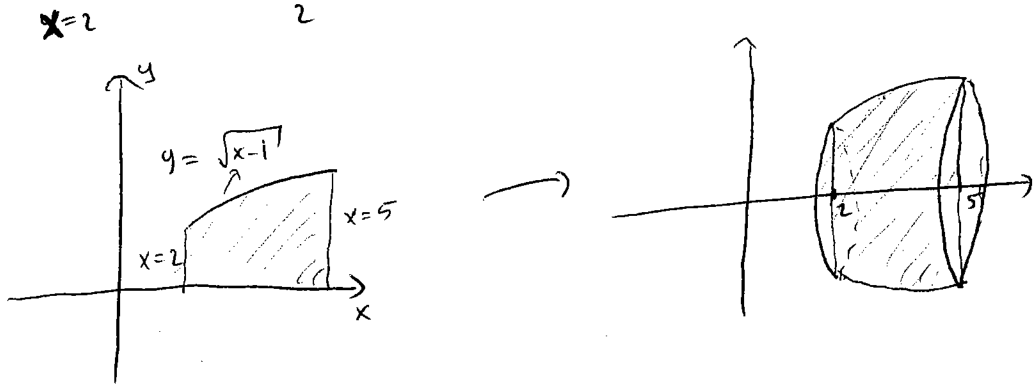
$$= \pi\left[x - \frac{1}{2}x^2 + \frac{1}{12}x^3\right]_0^2$$

$$= \frac{2}{3}\pi$$

4) $y = \sqrt{x-1}$, $x=2$, $x=5$, $y=0$; about the x-axis.

$$A(x) = \pi\left(\sqrt{x-1}\right)^2 = \pi(x-1)$$

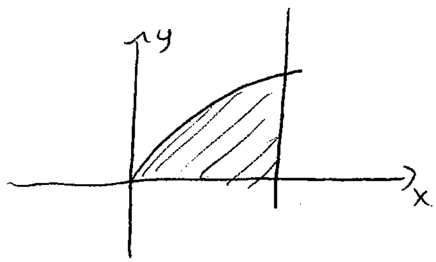
$$V = \int_{x=2}^{5} A(x)dx = \int_2^5 \pi(x-1)\,dx = \frac{15}{2}\pi$$

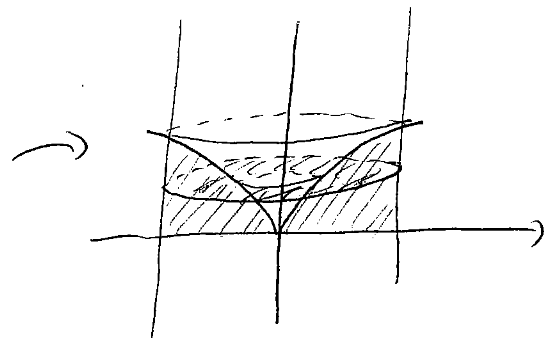


10) $y = x^{2/3} \iff x = y^{3/2}$ ②

$$A(y) = \pi (1)^2 - \pi \left(y^{3/2}\right)^2 = \pi(1-y^3)$$

$$V = \int_0^1 A(y)\,dy = \pi \int_0^1 (1-y^3)\,dy = \frac{3\pi}{4}$$

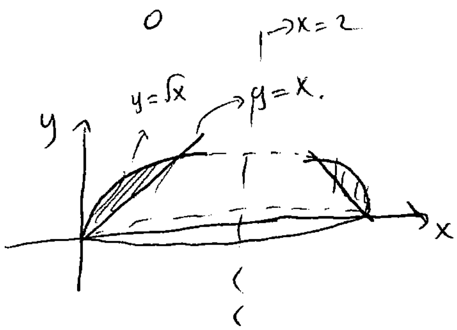




16) $y = \sqrt{x} \Rightarrow x = y^2$. So,

$$V = \int_0^1 \pi\left[(2-y^2)^2 - (2-y)^2\right]dy = \pi \int_0^1 (y^4 - 5y^2 + 4y)\,dy$$

$$= \pi\left(\frac{1}{5} - \frac{5}{3} + 2\right) = \frac{8}{15}\pi.$$



20) R_1 about OC, which is rotation about $x=0$

$$V = \int_0^1 A(y)\,dy = \int_0^1 \left[\pi 1^2 - \pi\left[y^{1/3}\right]^2\right]dy = \frac{2\pi}{5}$$

22) R_1 about the line $y=1$

$$V = \int_0^1 A(y)\,dy = \int_0^1 \pi\left(1 - y^{1/3}\right)^2 dy = \frac{\pi}{10}$$

24) R_2 about the line $x=0$,

$$V=\int_0^1 A(y)\,dy=\int_0^1 \pi(y^2)^2\,dy=\frac{\pi}{5}$$

26) R_2 about the line $y=1$

$$V=\int_0^1 A(x)\,dx=\int_0^1 \pi\left(1-\sqrt{x}\right)^2dx=\int_0^1 \pi\left(1-2x^{1/2}+x\right)dx$$

$$=\pi\left[x-\frac{4}{3}x^{3/2}+\frac{1}{2}x^2\right]_0^1=\frac{\pi}{6}$$

42) $\pi\int_2^5 y\,dy=\int_2^5 \pi.\left(\sqrt{y}\right)^2dy$ → describes the volume of the solid obtained by rotating $R=\{(x,y):\ 2\leq y\leq 5,\ 0\leq x\leq \sqrt{y}\}$. when we rotate this region around y-axis

44) $\int_0^1 \pi\left(y^4-y^8\right)dy=\int_{y=0}^{y=1}\left[\pi(y^2)^2-\pi(y^4)^2\right]dy$ → describes the volume of the solid obtained by rotation of $R=\{(x,y):\ 0\leq x\leq \frac{\pi}{2},\ 1\leq y\leq 1+\cos x\}$ of the xy-plane about the y-axis.

48) $$V=\pi\int_0^h\left(R-\frac{R-r}{h}y\right)^2dy=\pi\int_0^h\left(R^2-\frac{2R(R-r)}{h}y+\frac{(R-r)^2}{h^2}y^2\right)dy$$

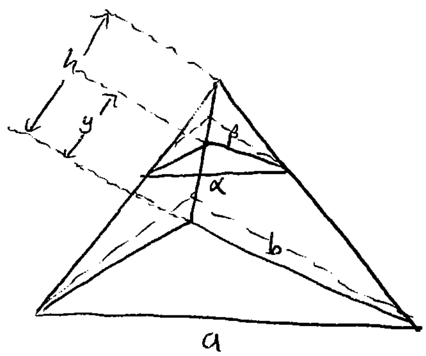
$$=\pi\left(R^2y-\frac{2R(R-r)}{h}\cdot\frac{y^2}{2}+\frac{(R-r)^2}{h^2}\cdot\frac{y^3}{3}\right)\Bigg|_0^h$$

$$=\pi\left(R^2h-\frac{2R(R-r).h^2}{2h}+\frac{(R-r)^2.h^3}{3h^2}\right)$$

$$= \pi\left(R^2h - 2R(R-r).h + \frac{(R-r)^2}{3h}\right)$$

39) $$V = \pi \int_0^{\pi} \left\{ \left[(\sin x)^2 - (-1)\right]^2 - \left[0-(-1)\right]^2 \right\} dx$$

$$= \frac{11}{8}\pi^2$$

52) 

By similar triangles, $\frac{a}{b} = \frac{\alpha}{\beta} \Rightarrow \alpha = \frac{a.\beta}{b}$

" " " , $\frac{b}{h} = \frac{\beta}{h-y}$

By these two equations $\alpha = a\left(1 - \frac{y}{h}\right)$

We've cross section as equilateral triangle hence

$$A(y) = \frac{1}{2}\,\alpha . \frac{\sqrt{3}}{2}\,\alpha = \frac{a^2\left(1 - y/h\right)^2 . \sqrt{3}}{4}$$

hence

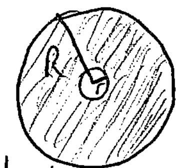
$$V = \int_0^h A(y)dy = \frac{a^2\sqrt{3}}{4}\int_0^h \left(1 - \frac{y}{h}\right)^2 dy = \frac{\sqrt{3}}{12}a^2h$$

63) a) Volume $(S_1) = \int_0^h A(z)dz =$ Volume (S_2) since at height z the cross section of solid has the same area $A(z)$

b) By Cavalieri's Principle, the volume of the cylinder in the figure is the same as that of a right circular cylinder with radius r and height h, that is, $\pi r^2 h$

66) <u>Case 1</u>: $0 \le h \le 10$. The ball will not be completely submerged

Then a cross sectional area at



height x above the bottom of the bowl by using Pythagorean thm can be found as follows:

$$R^2 = 15^2 - (15-x)^2$$

and

$$r^2 = 5^2 - (x-5)^2$$

$$A(x) = \pi(R^2 - r^2) = \pi\left[15^2 - (15-x)^2 - 5^2 + (x-5)^2\right]$$

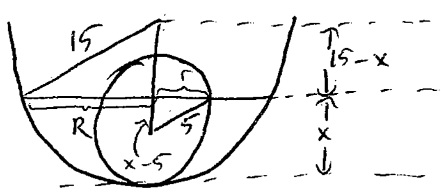
$$= \pi\left\{15^2 - 15^2 + 30x + x^2 - 5^2 + x^2 - 10x + 5^2\right\}$$

$$= 20\pi x$$

Volume at depth h is;

$$V(h) = \int_0^h A(x)dx = \int_0^h 20\pi x dx = 10\pi h^2 \text{ cm}^3 \quad 0 \le h \le 10$$

<u>Case 2</u>: $10 \le h \le 15$, the ball submerges.



Volume = subtracting the volume displaced by the ball from the total volume inside the bowl underneath the surface of the water. The total volume underneath the surface is just the volume of a cap of the bowl,

$$V_{cap}(h) = \frac{1}{3}\pi h^2(45-h)$$

$$V_{ball} = \frac{4}{3}\pi 5^3 = \frac{500}{3}\pi$$

So

$$V_{cap} - V_{ball} = \frac{1}{3}\pi\left(45h^2 - h^3 - 500\right)\text{cm}^3.$$