MATH 2423-010 Homework 1

1. Write out a complete proof of the formula for the sum on the first n squares

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

2. Read through your text, and find an expression for the sum of the first n cubes. Write this down (that's the hwk!)

3. Use the difference of adjacent 4th powers $(n+1)^4 - n^4$ to give a proof of the formula for the sum of cubes above.

4. Use limits of Riemann sums (as we're been doing in class notes, or as explained in sections 5.1 and 5.2 of the text) to give a careful calculation of the area under the graph $y = x^3$ between $x = 0$ and $x = 1$.

5. Write out the following without using summation notation (that is, write them out as long sums of many terms):

$$\sum_{i=1}^{5} \frac{1}{i+3}, \qquad \sum_{i=4}^{8} \frac{1}{i}, \qquad \sum_{i=10}^{14} \frac{1}{i-6}$$

6. The following two sums are Riemann sums for areas under functions over certain intervals on the x-axis. In each case, write down the function and the interval.

$$\sum_{i=1}^{n} \left(1 + \frac{i}{n}\right)\left(\frac{1}{n}\right), \qquad \sum_{i=1}^{n} \frac{1}{n\left(2 + \frac{i}{n}\right)^2}$$