Hwk 10

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(12) $\cosh x \cosh y + \sinh x \sinh y =$

$$\frac{1}{2}\left[e^{x}+e^{-x}\right]\cdot\frac{1}{2}\left[e^{y}+e^{-y}\right]+\frac{1}{2}\left[e^{x}-e^{-x}\right]\cdot\frac{1}{2}\left[e^{y}-e^{-y}\right]$$

$$=\frac{1}{4}\left[\left(e^{x+y}+\cancel{e^{x-y}}+\cancel{e^{-x+y}}+e^{-x-y}\right)+\left(e^{x+y}-\cancel{e^{x-y}}-\cancel{e^{-x+y}}+e^{-x-y}\right)\right]$$

$$=\frac{1}{4}\left[2e^{x+y}+2e^{-x-y}\right]=\frac{1}{2}\left(e^{x+y}+e^{-(x+y)}\right)$$

$$=\cosh(x+y)$$

(29(b)) Let $y=\tanh^{-1}x$. Then $x=\tanh y$.

(Use implicit diff.)

$1=\operatorname{sech}^2 y \, dy/dx \implies dy/dx=\dfrac{1}{\operatorname{sech}^2 y}=\dfrac{1}{1-\tanh^2 y}=\dfrac{1}{1-x^2}$

x

(36) $f(t)=e^t \operatorname{sech}(t)$

$f'(t)=e^t\left(-\operatorname{sech}(t)\tanh(t)\right)+\operatorname{sech}(t)\,e^t$

$\quad = e^t \operatorname{sech}(t)\left(1-\tanh(t)\right)$

Hwk 10 (cont.)

(40) $y = \sinh(\cosh x)$

$y' = \cosh(\cosh x) \cdot \sinh x$

(58) $\int \tanh x\, dx = \int \frac{\sinh x}{\cosh x}\, dx$ $\quad u = \cosh x$, $du = \sinh x\, dx$

$= \int \frac{1}{u}\, du$

$= \ln|u| + C = \ln|\cosh x| + C$

always positive

$= \ln(\cosh x) + C$

(62) $\int_0^1 \frac{dt}{\sqrt{16t^2+1}} = \frac{1}{4}\int_0^4 \frac{1}{\sqrt{u^2+1}}\, du = \frac{1}{4} \sinh^{-1} u \Big|_0^4$

$u = 4t$ $\quad t=0 \Rightarrow u=0$

$du = 4dt$ $\quad t=1 \Rightarrow u=4$

$= \frac{1}{4}(\sinh^{-1} 4 - \underbrace{\sinh^{-1}(0)}_{\to 0})$

$\boxed{= \frac{1}{4}(\sinh^{-1} 4)}$

OR

$= \frac{1}{4}(\ln(4+\sqrt{17}))$

from an identity in the book