HWK 12

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(10) $\int_0^{\pi} \cos^6\theta \, d\theta = \int_0^{\pi} (\cos^2\theta)^3 d\theta$

$= \frac{1}{8}\int_0^{\pi} (1+\cos 2\theta)^3 d\theta$

$= \frac{1}{8} \int_0^{\pi} (1 + 3\cos 2\theta + 3\cos^2 2\theta + \cos^3 2\theta) d\theta$

$= \frac{1}{8} \left[\theta \Big|_0^{\pi} + \frac{3}{2}\sin 2\theta \Big|_0^{\pi} + \frac{3}{2}\int_0^{\pi}(1+\cos 4\theta)d\theta + \int_0^{\pi}(1-\sin^2 2\theta)\cos 2\theta \, d\theta \right]$

$u = \sin 2\theta$ $\quad \cancel{x}=0 \rightarrow u=0$

$du = 2\cos 2\theta$ $\quad x=\pi \rightarrow u=0$

$= \frac{1}{8}\left[(\pi - 0) + \left(\frac{3}{2}\right)(\underset{0}{\sin 2\pi} - \underset{0}{\sin 0}) + \frac{3}{2}\left[\theta + \frac{1}{4}\sin 4\theta\right]_0^{\pi} + \frac{1}{2}\int_0^{0}(1-u^2)du\right]$

$= \frac{1}{8}\left[\pi \ \frac{3}{2}\left[\pi + \frac{1}{4}\underset{0}{\sin 4\pi} - (0 + \frac{1}{4}\underset{0}{\sin 0})\right] + 0\right]$

$= \frac{1}{8}\left[\pi + \frac{3}{2}\pi\right] = \boxed{\frac{5\pi}{16}}$

(22) $\int_0^{\pi/2} \sec^4(t/2)dt = 2\int_0^{\pi/4} \sec^4(u)du$

$= 2\int_0^{\pi/4} (\tan^2 u + 1)\sec^2 u \, du$

$u = t/2$ $\quad t=0 \rightarrow u=0$

$du = \frac{1}{2}t dt$ $\quad t=\pi/2 \rightarrow u=\pi/4$

$= 2\int_0^{1}(w^2+1)dw$

$= 2\left[\frac{w^3}{3} + w\right]_0^1$

$w = \tan u$ $\quad u=0 \rightarrow w=0$

$dw = \sec^2 u \, du$ $\quad u=\pi/4 \rightarrow w=1$

$= 2\left[\frac{1}{3} + 1 - (0+0)\right]$

$= \boxed{\frac{8}{3}}$

HWK 12 cont.

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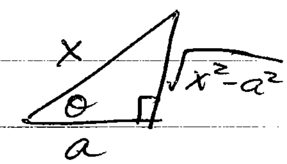
(8) $\displaystyle\int \frac{\sqrt{x^2-a^2}}{x^4}\,dx$

$x = a\sec\theta \longrightarrow \sec\theta = \dfrac{x}{a} \longrightarrow$ 

$dx = a\sec\theta\tan\theta\,d\theta$

$$\sqrt{x^2-a^2} = \sqrt{a^2\sec^2\theta - a^2} = a\sqrt{1-\sec^2\theta}$$
$$= a\sqrt{\tan^2\theta} = a\tan\theta$$

$$\int \frac{\sqrt{x^2-a^2}}{x^4}\,dx = \int \frac{a\tan\theta}{a^4\sec^4\theta}\cdot a\sec\theta\tan\theta\,d\theta$$
$$= \frac{1}{a^2}\int \frac{\tan^2\theta}{\sec^3\theta}\,d\theta$$
$$= \frac{1}{a^2}\int \frac{\sin^2\theta}{\cos^2\theta}\cdot\frac{\cos^3\theta}{1}\,d\theta$$
$$= \frac{1}{a^2}\int \sin^2\theta\cos\theta\,d\theta \qquad \boxed{\begin{aligned}u &= \sin\theta\\ du &= \cos\theta\,d\theta\end{aligned}}$$
$$= \frac{1}{a^2}\int u^2\,du$$
$$= \frac{1}{a^2}\,u^3 + C$$
$$= \frac{1}{a^2}\cdot\frac{1}{3}\sin^3\theta + C$$
$$= \frac{1}{3a^2}\left(\frac{\sqrt{x^2-a^2}}{x}\right)^3 + C$$

(26) $\displaystyle\int \frac{x^2}{\sqrt{4x-x^2}}\,dx$

1st we have to complete the square.

HWK 12 ctd.

(26) ctd.

$$4x - x^2 = -(x^2 - 4x + \underbrace{4}_{\left(\frac{-4}{2}\right)^2} - 4)$$

$$= -(x^2 - 4x + 4) + 4$$

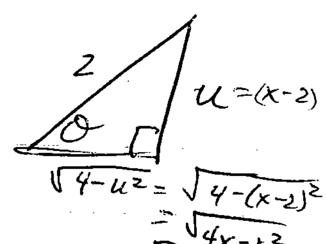
$$= -(x-2)^2 + 4 = 4 - (x-2)^2$$

$$\int \frac{x^2}{\sqrt{4x-x^2}}\,dx = \int \frac{x^2}{\sqrt{4-(x-2)^2}} = \int \frac{(u+2)^2}{\sqrt{4-u^2}}\,du$$

u-sub. $u = x-2 \rightarrow x = u+2.$
$du = dx$

Now we do a trig sub.

$$u = 2\sin\theta \rightarrow \sin\theta = \frac{u}{2} \longrightarrow$$



$$du = 2\cos\theta\,d\theta$$

$$\sqrt{4-u^2} = \sqrt{4-4\sin^2\theta} = 2\cos\theta$$

$$\int \frac{x^2}{\sqrt{4x^2-x^2}}\,dx = \int \frac{(2\sin\theta+2)^2}{\cancel{2\cos\theta}} \cdot \cancel{2\cos\theta}\,d\theta$$

$$= \int (2^2\sin^2\theta + 8\sin\theta + 4)\,d\theta$$

$$= 4 \cdot \tfrac{1}{2}\int(1-\cos 2\theta)\,d\theta + 8\int \sin\theta\,d\theta + \int 4\,d\theta$$

$$= 2\left[\theta - \tfrac{1}{2}\sin 2\theta\right] + 8(-\cos\theta) + 4\theta + C$$

use identity $\sin 2\theta = 2\sin\theta\cos\theta$

$$= 6\theta - \sin 2\theta - 8\cos\theta + C$$

$$= 6\theta - 2\sin\theta\cos\theta - 8\cos\theta + C$$

$$= 6\sin^{-1}\left(\frac{x-2}{2}\right) - 2\left(\frac{x-2}{2}\right)\left(\frac{\sqrt{4x-x^2}}{2}\right) - 8\left(\frac{\sqrt{4x-x^2}}{2}\right) + C$$

$$\boxed{= 6\sin^{-1}\left(\frac{x-2}{2}\right) - \frac{1}{2}(x-2)\sqrt{4x-x^2} - 4\sqrt{4x-x^2} + C}$$

From

$$\sin\theta = \frac{x-2}{2}$$

$$\cos\theta = \frac{\sqrt{4x-x^2}}{2}$$

$$\theta = \sin^{-1}\left(\frac{x-2}{2}\right)$$