$\mathbf{Q1}]\ldots [\mathbf{10} \ \mathbf{points}]$  Find and classify the critical points of the function

$$f(x,y) = x^4 + y^4 - 4xy$$

Q2]...[10 points] Use Lagrange multipliers to find the shortest distance from the origin to the surface  $xyz^2 = 2$ .

Q3]...[10 points] We are unable to anti-differentiate  $e^{-x^2}$ . However, we can still evaluate the double integral

$$\int_0^1 \int_y^1 e^{-x^2} \, dx \, dy$$

Show all the steps involved in evaluating this double integral.

Q4]...[10 points] Use double integrals to find the area of the portion of the conical surface  $3z^2 = x^2 + y^2$  where  $1 \le z \le 2$ .

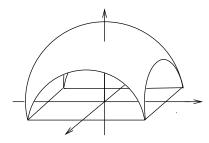
Q5]...[10 points] Consider the triple integral

$$\int_0^1 \int_z^1 \int_0^{x-z} f(x,y,z) \, dy dx dz$$

Sketch the projections of the region of integration on the three coordinate planes.

Rewrite the integral so that the outermost integral is with respect to x and the innermost integral is with respect to z.

**Bonus**]... Find the surface area of the portion of the sphere  $x^2 + y^2 + z^2 = 2$  of radius  $\sqrt{2}$  which lies above the square with vertices  $(\pm 1, \pm 1)$  in the *xy*-plane.



Rough Work Page