Hwk I

Allowed to use:

$$|A \cup B| = |A| + |B| - |A \cap B| \qquad \text{— ①}$$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| \qquad \text{— ②}$$

Find expression for $|A \cup B \cup C \cup D|$ + give proof.

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one way (only uses ① !)

Let $A \cup B = P \qquad C \cup D = Q$

Then

$$|A \cup B \cup C \cup D| = |P \cup Q|$$

$$= |P| + |Q| - |P \cap Q| \quad \text{--- by ①}$$

$$= |A \cup B| + |C \cup D| - |(A \cup B) \cap Q|$$

$$= |A| + |B| - |A \cap B| + |C| + |D| - |C \cap D|$$
$$- |(A \cap Q) \cup (B \cap Q)| \quad \text{— by ① twice \& distrib law } \cap \text{ over } \cup$$

$$= |A| + |B| + |C| + |D| - |A \cap B| - |C \cap D|$$
$$- \left(|A \cap Q| + |B \cap Q| - |A \cap Q \cap B \cap Q| \right) \quad \text{-- by ①}$$

$$= |A| + |B| + |C| + |D| - |A \cap B| - |C \cap D|$$
$$- |A \cap (C \cup D)| - |B \cap (C \cup D)| + |A \cap B \cap Q|$$

$$= |A| + |B| + |C| + |D| - |A\cap B| - |C\cap D|$$
$$- |(A\cap C) \cup (A\cap D)| - |(B\cap C) \cup (B\cap D)|$$
$$- |A\cap B \cap (C\cup D)|$$ --- distrib laws thrice

$$= |A| + |B| + |C| + |D| - |A\cap B| - |C\cap D|$$
$$- \left(|A\cap C| + |A\cap D| - |A\cap C \cap A\cap D|\right)$$
$$- \left(|B\cap C| + |B\cap D| - |B\cap C \cap B\cap D|\right)$$ --- by (1) twice
$$+ |(A\cap B\cap C) \cup (A\cap B\cap D)|$$ --- by distrib law

$$= |A| + |B| + |C| + |D| - |A\cap B| - |C\cap D|$$
$$- |A\cap C| - |A\cap D| + |A\cap C\cap D|$$
$$- |B\cap C| - |B\cap D| + |B\cap C\cap D|$$
$$+ \left(|A\cap B\cap C| + |A\cap B\cap D| - |A\cap B\cap C \cap A\cap B\cap D|\right)$$
--- by (1) again

$$= |A| + |B| + |C| + |D| - |A\cap B| - |A\cap C| - |A\cap D| - |B\cap C| - |B\cap D| - |C\cap D|$$
$$+ |A\cap C\cap D| + |A\cap B\cap D| + |A\cap B\cap C| + |B\cap C\cap D| - |A\cap B\cap C\cap D|$$ (*)

Another way .... (uses ① & ②)   Let $A \cup B \cup C = P$

$$|A \cup B \cup C \cup D| = |P \cup D|$$

$$= |P| + |D| - |P \cap D| \quad \text{—— by ①}$$

$$= |A \cup B \cup C| + |D| - |(A \cup B \cup C) \cap D|$$

$$= |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| \quad \text{--- by ②}$$

$$+ |D| - |(A \cap D) \cup (B \cap D) \cup (C \cap D)| \quad \text{--- by distrib. laws.}$$

$$= |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |B \cap C|$$

$$- \Big(|A \cap D| + |B \cap D| + |C \cap D| - |A \cap D \cap B \cap D|$$

$$- |A \cap D \cap C \cap D| - |B \cap D \cap C \cap D| + |A \cap D \cap B \cap D \cap C \cap D| \Big)$$

--- by ②

$$= |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D|$$

$$- |C \cap D| + |A \cap B \cap C| + |A \cap B \cap D| + |A \cap C \cap D| + |B \cap C \cap D|$$

$$- |A \cap B \cap C \cap D|$$

—— (*) again !

Using ① & ② is any easier way !