MATH 4513-001 Senior Seminar Spring 2010

Homework Set 3

Please complete by class time on Thursday, Feb 25.

1. Write down five different elements $g \in S_5$ which conjugate $(12)(34)$ into $(13)(24)$. That is, find five different elements $g \in S_5$ which satisfy the equation

$$g(12)(34)g^{-1} = (13)(24)$$

2. Write down a detailed argument to show that the $(n-1)n/2$ transpositions (pq) for $1 \leq p < q \leq n$ generate all of S_n.

3. Write down a detailed argument to show that the two elements (12) and $(1 \ldots n)$ generate all of S_n.

4. Verify that $\{(12), (123)\}$, $\{(12), (23)\}$ are two generating sets for S_3. Also, draw the Cayley graphs of S_3 with respect to these two generating sets. You should draw two separate graphs.

5. Compare the Cayley graph of S_3 with respect to $\{(12), (123)\}$ with the Cayley graph of $\mathbb{Z}_6$ with respect to $\{2, 3\}$. Any similarities? Any differences?

6. Draw the Cayley graph of S_4 with respect to the generating set $\{(12), (23), (34)\}$ (also verify that this is indeed a generating set).