pge 969 Q 52 (Q53 similar)

Let $F(x,y,z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$ ellipsoid is $F(x,y,z) = 1$ level set.

Normal vector $= \nabla F(x_0, y_0, z_0)$

$$= \left\langle \frac{2x_0}{a^2}, \frac{2y_0}{b^2}, \frac{2z_0}{c^2} \right\rangle$$

Point $= (x_0, y_0, z_0)$

Equation of T. Plane $\Rightarrow$ $\left\langle \frac{2x_0}{a^2}, \frac{2y_0}{b^2}, \frac{2z_0}{c^2} \right\rangle \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0$

$$2\left[\frac{xx_0 - x_0^2}{a^2} + \frac{yy_0 - y_0^2}{b^2} + \frac{zz_0 - z_0^2}{c^2}\right] = 0$$

$$\Rightarrow \frac{xx_0}{a^2} + \frac{yy_0}{b^2} + \frac{zz_0}{c^2} = \frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} + \frac{z_0^2}{c^2} = 1$$

$F(x_0, y_0, z_0) = 1$

$$\boxed{\frac{xx_0}{a^2} + \frac{yy_0}{b^2} + \frac{zz_0}{c^2} = 1}$$

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pge 978 Q 14 $f(x,y) = y\cos x$

$$\left.\begin{aligned} f_x &= -y\sin x \\ f_y &= \cos x \end{aligned}\right)$$

$$\left.\begin{aligned} f_x &= 0 \\ f_y &= 0 \end{aligned}\right) \Rightarrow \left.\begin{aligned} &\cos x = 0 \\ &\& \\ &y\sin x = 0 \end{aligned}\right)$$

$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

$$\boxed{x = \frac{\pi}{2} + n\pi \quad n \text{ an integer}}$$

$\sin(x) = \pm 1 \neq 0$

$\Rightarrow$ 2nd eqn gives $\boxed{y = 0}$

C.P.'s $\left(\frac{\pi}{2} + n\pi, 0\right)$ $n = 0, \pm 1, \pm 2, \ldots$

$$D = f_{xx} f_{yy} - (f_{xy})^2 = (-y\cos x)(0) - (-\sin x)^2$$
$$= -\sin^2 x.$$
$$= -(\pm 1)^2$$
$$= -1$$
$$< 0$$

$\Rightarrow$ SADDLES at all these points

Q4 $y^2 = 9 + xz$ closest to $(0,0,0)$

Minimize square of distance from $(0,0,0)$ to (x,y,z) subject to $y^2 = 9 + xz$.

Minimize $(x-0)^2 + (y-0)^2 + (z-0)^2$
$= x^2 + y^2 + z^2$

subject to $y^2 = 9 + xz$

$$\boxed{\text{Minimize } f(x,z) = x^2 + (9 + xz) + z^2.}$$ Reformulation

$f_x = 2x + z$

$f_z = x + 2z$

$\left.\begin{array}{l} f_x = 0 \\ f_z = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} 2x + z = 0 \\ x + 2z = 0 \end{array}\right.$

$z = -2x$

$x + 2(-2x) = 0$

$-3x = 0 \Rightarrow x = 0$ $\Rightarrow z = 0$

C.P. $= (0,0)$

Check $\quad f_{xx} f_{zz} - (f_{xz})^2 = (2)(2) - (1)^2 = 3 > 0$

$\quad \& \; f_{xx} = 2 > 0$

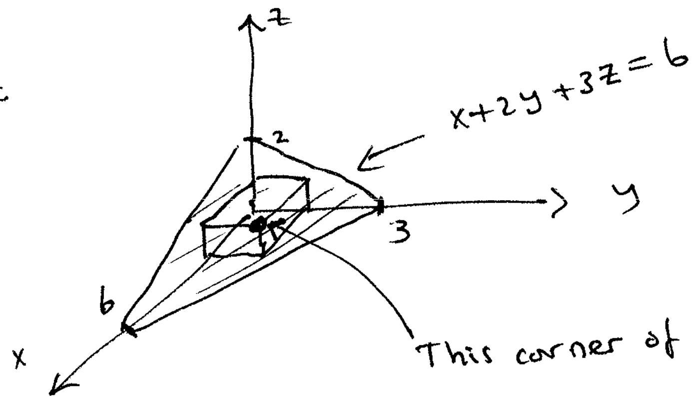
$\quad \Rightarrow$ local ($\&$ global) min!

$y^2 = 9 + (0)(0) \qquad y^2 = 9 \qquad y = \pm 3$

Points on surface are $(0,0,3)$ (and $(0,0,-3)$)

& they are <u>3 units</u> from $(0,0,0)$

Q47



This corner of the box (x,y,z) lies on the plane.

$\text{Vol} = (x)(y)(z)$.

Problem $\quad$ Maximize $V = xyz$ $\quad$ subject to $x + 2y + 3z = 6$

Maximize $\boxed{V(y,z) = (6 - 2y - 3z)\,y\,z}$ $\qquad$ Reformulation.

$$\left.\begin{array}{l} V_y = 0 \\ V_z = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} (6-2y-3z)z - 2yz = 0 \\ (6-2y-3z)y - 3yz = 0 \end{array}\right.$$

$$(6 - 2y - 3z - 2y)\, z = 0$$
$$(6 - 2y - 3z - 3z)\, y = 0$$

$y = 0$
$z = 0$ not physically meaningful!

$$6 - 4y - 3z = 0$$
$$6 - 2y - 6z = 0$$

$$12 - 4y - 12z = 0$$
Subtract
$$6 - 4y - 3z = 0$$

$$6 - 9z = 0 \qquad z = \frac{6}{9} = \frac{2}{3}$$

$\boxed{z = \frac{2}{3}}$

$$6 - 4y - 3\left(\frac{2}{3}\right) = 0$$

$4y = 4$
$\boxed{y = 1}$

$$x = 6 - 2y - 3z$$
$$= 6 - 2(1) - 3\left(\frac{2}{3}\right)$$
$$= 6 - 2 - 2 = 2.$$

$\boxed{x = 2}$

Ans: $\boxed{(x, y, z) = \left(2, 1, \frac{2}{3}\right) \quad \& \quad \text{Vol} = \frac{4}{3}}$