

Feb 06 2015

Midterm I

50mins

Name:

Student ID:

**Instructions.**

1. Attempt all questions.
2. Do not write on back of exam sheets. Extra paper is available if you need it.
3. Show all the steps of your work clearly.
4. No calculators, notes, books etc.

Question	Points	Your Score
Q1	30	
Q2	20	
Q3	20	
Q4	30	
TOTAL	100	

**Q1]... [30 points]** (a) What two pieces of geometric information do you need to uniquely specify a plane in 3-dimensional space?

(b) Use vectors to find the equation of the plane containing the points  $(1, 2, 3)$ ,  $(1, 0, 1)$  and  $(0, 2, 1)$ . Show all the steps of your work.

(c) Use vectors to find the distance from the point  $(1, 1, 1)$  to the plane  $x + 2y - z = 0$ .

**Q2]... [20 points]** a) What two pieces of geometric information do you need to uniquely specify a line in 3-dimensional space?

Find the equation of the **tangent line** to the curve  $\mathbf{r}(t) = \langle t, t^2 + 1, t^3 + t \rangle$  at the point where the parameter  $t = 1$ . Show all the steps of your work.

**Q3]... [20 points]** A stream of air is flowing uniformly through space with constant velocity vector  $\mathbf{v} = \langle 1, 2, 3 \rangle$  feet per second. Find the volume of air which flows through a parallelogram region with sides  $\langle 1, 0, 2 \rangle$  and  $\langle 2, 2, 1 \rangle$  feet in 2 seconds.

**Q4]... [30 points]** (a) Consider the function of several variables

$$f(x, y) = 4x^2 - y^2$$

Draw the following **level curves**:  $f = 0$ ;  $f = 1$ ;  $f = -1$ ;  $f = 4$ ;  $f = -4$ .

(b) Describe the shape of the graph of the function  $f(x, y) = 4x^2 - y^2$  above.

(c) Describe the surface  $y = z^2$  in 3-dimensional space.

**Bonus Problem.** Suppose that the point  $(x_1, y_1, z_1)$  does not lie on the line  $L$  given by the vector equation

$$\langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t\langle v_1, v_2, v_3 \rangle$$

Describe the steps needed (using cross products and dot products of vectors) to find the distance from the point  $(x_1, y_1, z_1)$  to the line  $L$ . Give details of your reasoning.